Rational Expressions & Functions

Name: _____ Class: _____ Date: _____

Use this packet alongside the Unit 5 website. Complete notes and examples with your teacher, work through practice, then complete the skills check independently before moving to the next lesson.

LESSON SEQUENCE

5.1	Simplifying Rational Expressions	pp. 2–4
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UNIT REFERENCE / Record restrictions before you simplify

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$y = \frac{a}{x - h} + k$$

Vocabulary preview: rational expression, LCD, reciprocal, complex fraction, extraneous solution, domain, asymptote, hole.

After the lessons: cumulative review (pp. 28–30) and Waterworks engineering investigation (pp. 31–33).

Course website: welchteachingandcoaching.com/algebra2/unit5

Simplifying Rational Expressions

Factor first, cancel common factors, and keep restrictions from the original denominator.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Factor before canceling

Original-denominator restriction

I ILLUSTRATION • Use a diagram or annotated expression.

Sketch or label numerator, denominator, and common factors. Show why terms cannot be canceled across addition.

D DEFINITIONS • Write a definition for each term.

Rational expression:

Domain restriction:

Common factor:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$\frac{12x^3}{18x^2}$$

Factor the monomials; reduce the coefficient and powers.

Example 2

$$\frac{x^2+5x+6}{x^2+4x+3}$$

Factor both polynomials. List restrictions before canceling.

Simplifying Rational Expressions

Show each step. Simplify fully and state all original restrictions.

GUIDED / 1

$$\frac{15x^4}{20x^2}$$

Work:

Answer: _____

Restrictions: _____

GUIDED / 2

$$\frac{x^2-16}{x^2+7x+12}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 3

$$\frac{18a^3b}{24ab^2}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 4

$$\frac{x^2+7x+10}{x^2+3x-10}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 5

$$\frac{3x^2-12x}{6x^2}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 6

$$\frac{x^2-25}{x^2-10x+25}$$

Work:

Answer: _____

Restrictions: _____

Skills Check 5.1

Simplifying Rational Expressions • Complete independently. Show work, state restrictions, and box your answer.

Name: _____ Date: _____

QUESTION 1

$$\frac{x^2+2x-15}{x^2-9}$$

Restrictions: _____

Show your work:

Final answer: _____

QUESTION 2

$$\frac{4x^2-16x}{2x^2-8x}$$

Restrictions: _____

Show your work:

Final answer: _____

Multiplying & Dividing

Factor every expression. Rewrite division as multiplication by the reciprocal; exclude zeros of the divisor.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Multiply numerators and denominators

Nonzero divisor and original restrictions

I ILLUSTRATION • Use a diagram or annotated expression.

Draw arrows to show how a divisor becomes its reciprocal. Circle original denominators and the divisor numerator.

D DEFINITIONS • Write a definition for each term.

Reciprocal:

Product:

Quotient:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$\frac{x^2-9}{x^2+2x-3} \cdot \frac{x+1}{x-3}$$

Factor all expressions; cancel common factors.

Example 2

$$\frac{x^2-4}{x+3} \div \frac{x-2}{x+3}$$

Identify restrictions, invert the divisor, then multiply.

Multiplying & Dividing

Show each step. Simplify fully and state all original restrictions.

GUIDED / 1

$$\frac{6x^2}{5x+10} \cdot \frac{x+2}{9x}$$

Work:

Answer: _____

Restrictions: _____

GUIDED / 2

$$\frac{x^2-9}{x+4} \div \frac{x-3}{x+4}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 3

$$\frac{x^2-25}{x+2} \cdot \frac{x+2}{x+5}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 4

$$\frac{3x}{x-1} \div \frac{6x^2}{x-1}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 5

$$\frac{x^2+5x+6}{x^2-4} \cdot \frac{x-2}{x+3}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 6

$$\frac{x^2-1}{x^2+3x+2} \div \frac{x-1}{x+2}$$

Work:

Answer: _____

Restrictions: _____

Skills Check 5.2

Multiplying & Dividing • Complete independently. Show work, state restrictions, and box your answer.

Name: _____ Date: _____

QUESTION 1

$$\frac{x^2-16}{x+1} \div \frac{x-4}{x+1}$$

Restrictions: _____

Show your work:

Final answer: _____

QUESTION 2

$$\frac{x^2-9}{x^2+x-6} \cdot \frac{x+3}{x-3}$$

Restrictions: _____

Show your work:

Final answer: _____

Adding & Subtracting | Day 1

Build a common denominator, rewrite equivalent fractions, and combine numerators.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Common denominator rule

Combine numerators only

I ILLUSTRATION • Use a diagram or annotated expression.

Make a fraction model or factor diagram showing how to find the LCD when one denominator divides the other.

D DEFINITIONS • Write a definition for each term.

Least common denominator (LCD):

Equivalent fractions:

Numerator:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$\frac{2x+1}{x-4}+\frac{x-5}{x-4}$$

Combine numerators over the common denominator.

Example 2

$$\frac{2}{x(x+1)}-\frac{1}{x+1}$$

Find the LCD, rewrite the second fraction, then subtract.

Adding & Subtracting | Day 1

Show each step. Simplify fully and state all original restrictions.

GUIDED / 1

$$\frac{3}{x} + \frac{2}{x}$$

Work:

Answer: _____

Restrictions: _____

GUIDED / 2

$$\frac{5}{x-2} - \frac{x}{x-2}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 3

$$\frac{1}{x} + \frac{2}{x+3}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 4

$$\frac{4}{x(x+2)} - \frac{1}{x+2}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 5

$$\frac{2}{3x} + \frac{5}{6x}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 6

$$\frac{x+1}{x+4} - \frac{3}{x+4}$$

Work:

Answer: _____

Restrictions: _____

Skills Check 5.3

Adding & Subtracting | Day 1 • Complete independently. Show work, state restrictions, and box your answer.

Name: _____ Date: _____

QUESTION 1

$$\frac{2}{x} + \frac{1}{x-4}$$

Restrictions: _____

Show your work:

Final answer: _____

QUESTION 2

$$\frac{5}{x(x+3)} - \frac{2}{x+3}$$

Restrictions: _____

Show your work:

Final answer: _____

Adding & Subtracting | Day 2

Factor polynomial denominators, build the LCD using highest powers, and distribute subtraction.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Highest-power LCD rule

Distribute the subtraction sign

I ILLUSTRATION • Use a diagram or annotated expression.

Use a factor organizer to compare $(x-2)(x+2)$ and $(x-2)^2$. Show which factors the LCD needs.

D DEFINITIONS • Write a definition for each term.

Polynomial denominator:

Distinct factor:

Least common denominator:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$\frac{1}{x^2-1}+\frac{2}{x+1}$$

Factor the quadratic; rewrite the second numerator.

Example 2

$$\frac{x+2}{x-3}-\frac{2x-1}{x-3}$$

Use parentheses for the entire numerator being subtracted.

Adding & Subtracting | Day 2

Show each step. Simplify fully and state all original restrictions.

GUIDED / 1

$$\frac{2}{x^2-9}+\frac{1}{x+3}$$

Work:

Answer: _____

Restrictions: _____

GUIDED / 2

$$\frac{3}{x^2-4}-\frac{1}{x-2}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 3

$$\frac{1}{(x-1)^2}+\frac{2}{x-1}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 4

$$\frac{x}{x^2+x-6}-\frac{1}{x+3}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 5

$$\frac{2}{x^2-4}+\frac{3}{x^2+4x+4}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 6

$$\frac{x+1}{x-2}-\frac{2x-3}{x+2}$$

Work:

Answer: _____

Restrictions: _____

Skills Check 5.4

Adding & Subtracting | Day 2 • Complete independently. Show work, state restrictions, and box your answer.

Name: _____ Date: _____

QUESTION 1

$$\frac{1}{x^2-4} - \frac{1}{x-2}$$

Restrictions: _____

Show your work:

Final answer: _____

QUESTION 2

$$\frac{2}{x^2+5x+6} + \frac{3}{x+2}$$

Restrictions: _____

Show your work:

Final answer: _____

Simplifying Complex Fractions

Clear inner denominators using an LCD; exclude any value making the entire denominator zero.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Multiply top and bottom by the LCD

Entire denominator must be nonzero

I ILLUSTRATION • Use a diagram or annotated expression.

Label the top expression, bottom expression and inner denominators in a nested fraction.

D DEFINITIONS • Write a definition for each term.

Complex fraction:

Inner denominator:

Divisor:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}}$$

Find the numerator first; divide by the denominator.

Example 2

$$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x}}$$

Multiply top and bottom by 2x; state restrictions.

Simplifying Complex Fractions

Show each step. Simplify fully and state all original restrictions.

GUIDED / 1

$$\frac{\frac{1}{3}+\frac{1}{6}}{\frac{1}{2}}$$

Work:

Answer: _____

Restrictions: _____

GUIDED / 2

$$\frac{\frac{1}{x}+\frac{1}{4}}{\frac{1}{x}}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 3

$$\frac{\frac{2}{x}-\frac{1}{3}}{\frac{1}{x}}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 4

$$\frac{\frac{1}{x}+\frac{1}{2}}{\frac{1}{x}-\frac{1}{2}}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 5

$$\frac{\frac{3}{x}}{\frac{2}{x}+\frac{1}{4}}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 6

$$\frac{\frac{1}{x-1}+\frac{1}{x+1}}{\frac{2}{x^2-1}}$$

Work:

Answer: _____

Restrictions: _____

Skills Check 5.5

Simplifying Complex Fractions • Complete independently. Show work, state restrictions, and box your answer.

Name: _____ Date: _____

QUESTION 1

$$\frac{\frac{1}{x}+\frac{1}{5}}{\frac{1}{x}}$$

Restrictions: _____

Show your work:

Final answer: _____

QUESTION 2

$$\frac{\frac{1}{x}+\frac{1}{3}}{\frac{1}{x}-\frac{1}{3}}$$

Restrictions: _____

Show your work:

Final answer: _____

Solving Rational Equations

State restrictions, clear denominators with the LCD, solve, and check in the original equation.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Multiply every term by the LCD

Check in the original equation

I ILLUSTRATION • Use a diagram or annotated expression.

Create a flowchart: restrictions → clear denominators → solve → check → solution set.

D DEFINITIONS • Write a definition for each term.

Rational equation:

Candidate solution:

Extraneous solution:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$\frac{3}{x-1} = \frac{1}{2}$$

State restrictions, multiply by 2(x−1), solve and check.

Example 2

$$\frac{x}{x-2} = \frac{2}{x-2}$$

Explain why the candidate is excluded.

Solving Rational Equations

Show each step. Solve and verify candidates in the original equation.

GUIDED / 1

$$\frac{4}{x+2} = \frac{1}{3}$$

Work:

Answer: _____

Restrictions: _____

GUIDED / 2

$$\frac{1}{x} + \frac{1}{2} = 1$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 3

$$\frac{2}{x-3} = \frac{5}{x+1}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 4

$$\frac{x}{x-4} = \frac{4}{x-4}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 5

$$\frac{1}{x} + \frac{1}{x+2} = \frac{3}{x(x+2)}$$

Work:

Answer: _____

Restrictions: _____

INDEPENDENT / 6

$$\frac{x+1}{x-2} = \frac{3}{x-2}$$

Work:

Answer: _____

Restrictions: _____

Skills Check 5.6

Solving Rational Equations • Complete independently. Show work, state restrictions, and box your answer.

Name: _____ Date: _____

QUESTION 1

$$\frac{2}{x-1} = \frac{1}{3}$$

Restrictions: _____

Show your work:

Final answer: _____

QUESTION 2

$$\frac{x+2}{x-3} = \frac{5}{x-3}$$

Restrictions: _____

Show your work:

Final answer: _____

Graphing Rational Functions | Day 1

Graph translated reciprocal functions using asymptotes and points on both sides of the vertical asymptote.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Asymptotes of y = a/(x − h) + k

Graph both branches using a table

I ILLUSTRATION • Use a diagram or annotated expression.

Sketch the parent y = 1/x. Label its asymptotes and show the direction of the two branches.

D DEFINITIONS • Write a definition for each term.

Vertical asymptote:

Horizontal asymptote:

Domain and range:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$y = \frac{1}{x-2} + 1$$

Identify h, k, VA, HA; choose points on both sides.

Example 2

$$y = \frac{-2}{x+1} - 1$$

Describe the shifts and reflection; sketch the graph.

Graphing Rational Functions | Day 1

State domain, asymptotes and hole(s), if any. Sketch the function and label its features.

PROBLEM 1

$$y = \frac{1}{x+2} - 1$$

Domain:

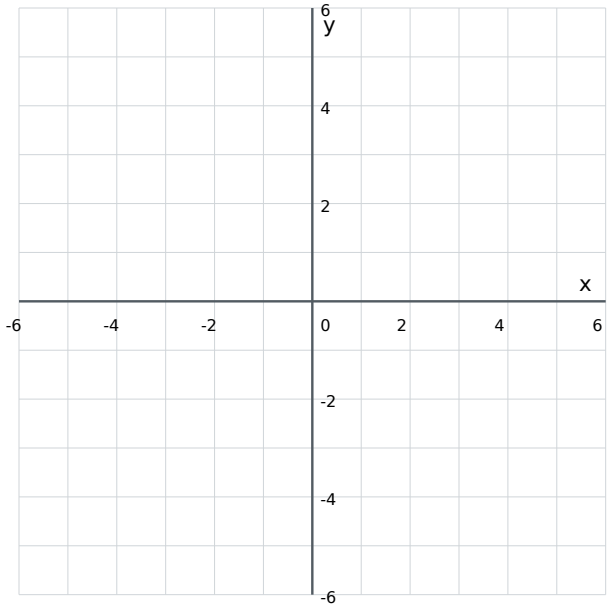
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



PROBLEM 2

$$y = \frac{2}{x-1} + 2$$

Domain:

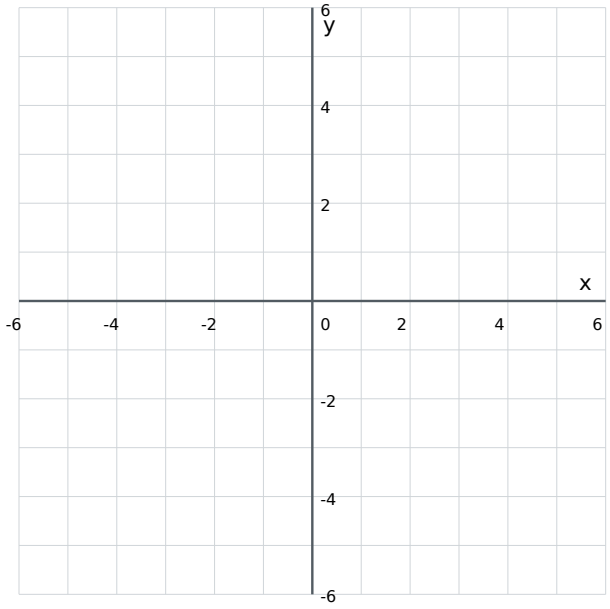
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



Graphing Rational Functions | Day 1

State domain, asymptotes and hole(s), if any. Sketch the function and label its features.

PROBLEM 3

$$y = \frac{-1}{x-3}$$

Domain:

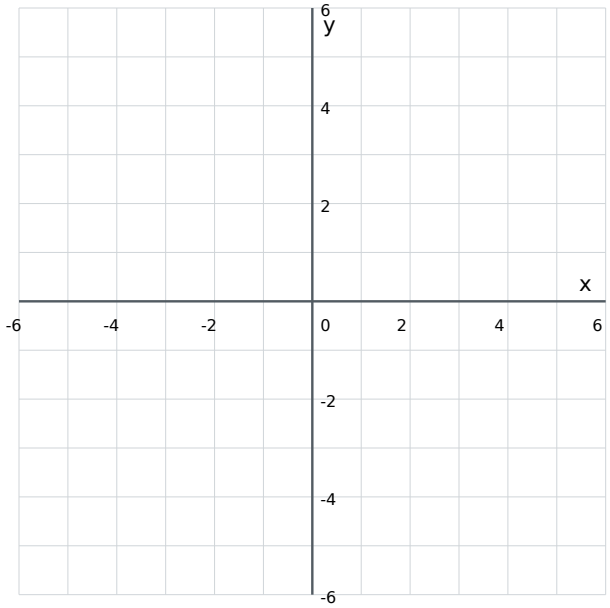
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



PROBLEM 4

$$y = \frac{2}{x+1} - 2$$

Domain:

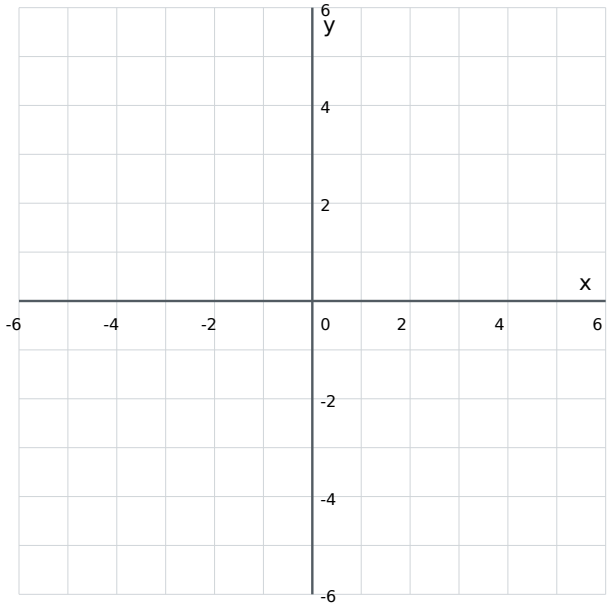
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



Skills Check 5.7A

Complete independently. State domain, asymptotes and hole(s), if any. Sketch the function and label its features.

QUESTION 1

$$y = \frac{1}{x+4} + 2$$

Domain:

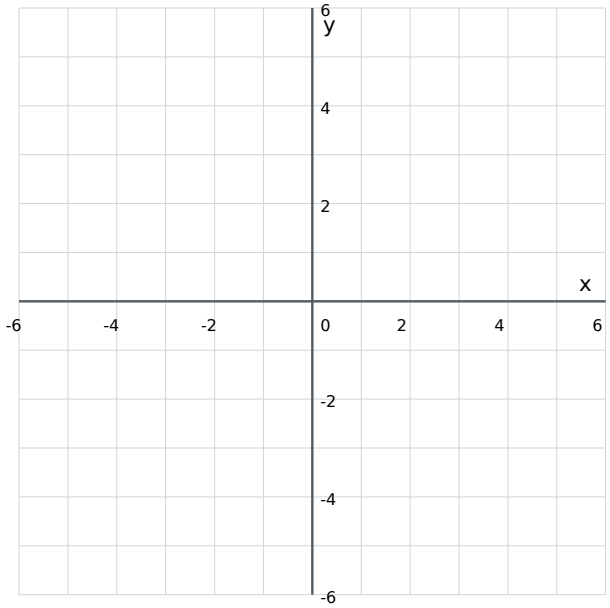
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



QUESTION 2

$$y = \frac{-2}{x-2} - 1$$

Domain:

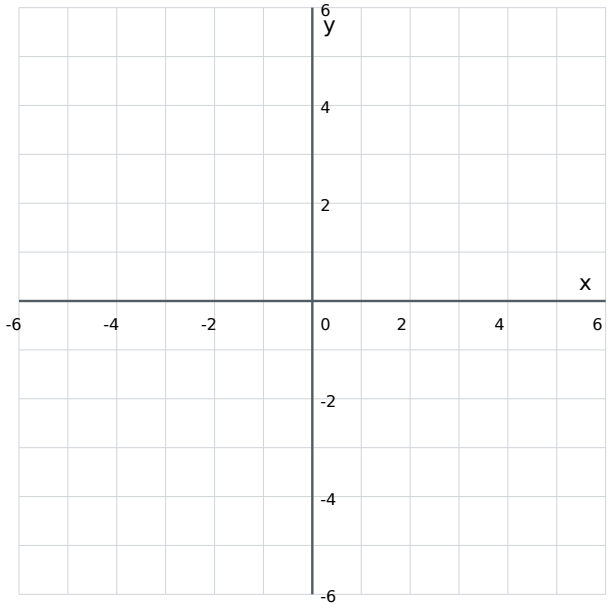
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



Graphing Rational Functions | Day 2

Factor, identify original exclusions, distinguish holes from vertical asymptotes, and find horizontal asymptotes.

S SUBJECT • What are we learning?

R RULES • Explain each rule in your own words.

Canceled factor and hole rule

Remaining denominator zero and degree comparison

I ILLUSTRATION • Use a diagram or annotated expression.

Sketch a curve with a hole and a vertical asymptote. Label the excluded x-values and explain the difference.

D DEFINITIONS • Write a definition for each term.

Removable discontinuity (hole):

End behavior:

Intercept:

E / S EXAMPLES & STEPS • Work with your teacher.

Example 1

$$y = \frac{x-3}{(x-2)(x-3)}$$

Simplify; find the hole, VA and HA.

Example 2

$$y = \frac{x^2-1}{x^2-3x+2}$$

Factor; find hole, asymptotes and intercepts.

Graphing Rational Functions | Day 2

State domain, asymptotes and hole(s), if any. Sketch the function and label its features.

PROBLEM 1

$$y = \frac{x+1}{x^2+3x+2}$$

Domain:

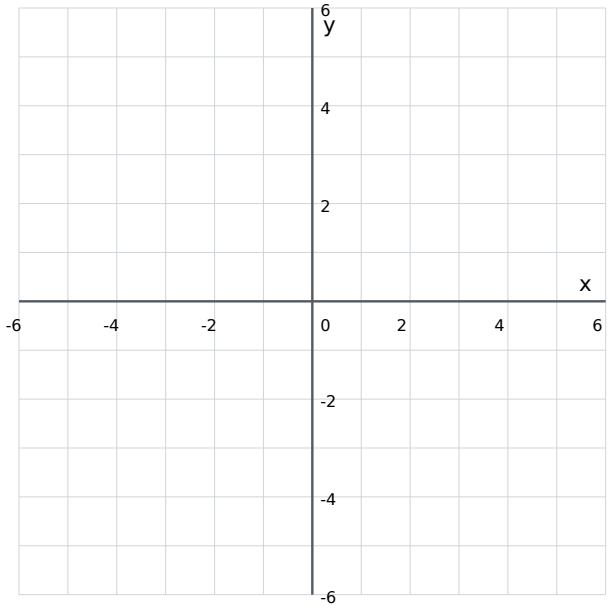
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



PROBLEM 2

$$y = \frac{x^2-4}{x^2-x-2}$$

Domain:

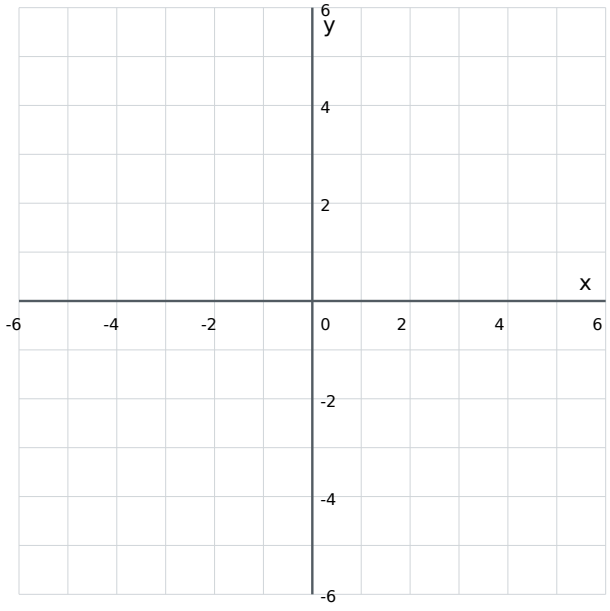
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



Graphing Rational Functions | Day 2

State domain, asymptotes and hole(s), if any. Sketch the function and label its features.

PROBLEM 3

$$y = \frac{x^2 - 9}{x - 3}$$

Domain:

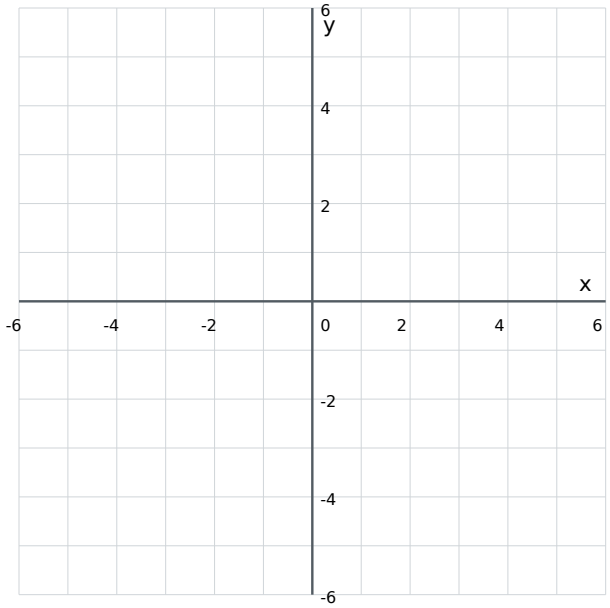
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



PROBLEM 4

$$y = \frac{2x + 1}{x - 2}$$

Domain:

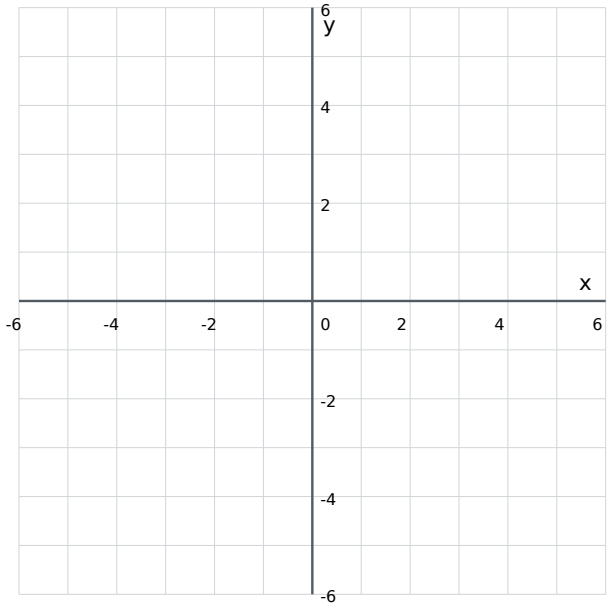
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



Skills Check 5.7B

Complete independently. State domain, asymptotes and hole(s), if any. Sketch the function and label its features.

QUESTION 1

$$y = \frac{x-2}{(x-2)(x+3)}$$

Domain:

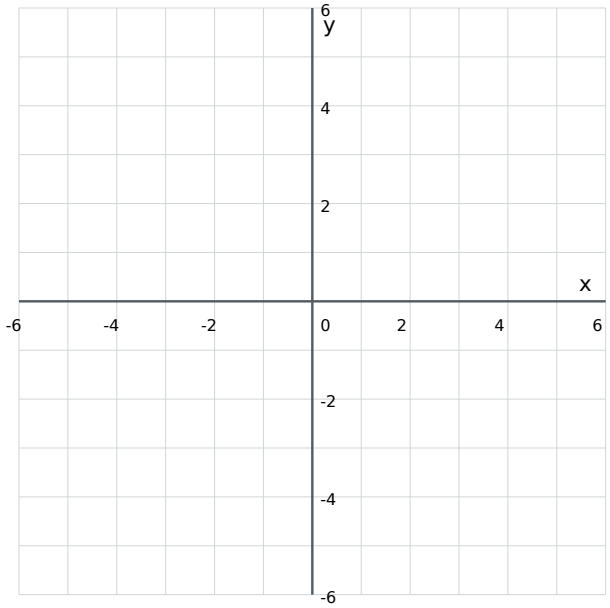
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



QUESTION 2

$$y = \frac{x^2-4}{x^2-3x+2}$$

Domain:

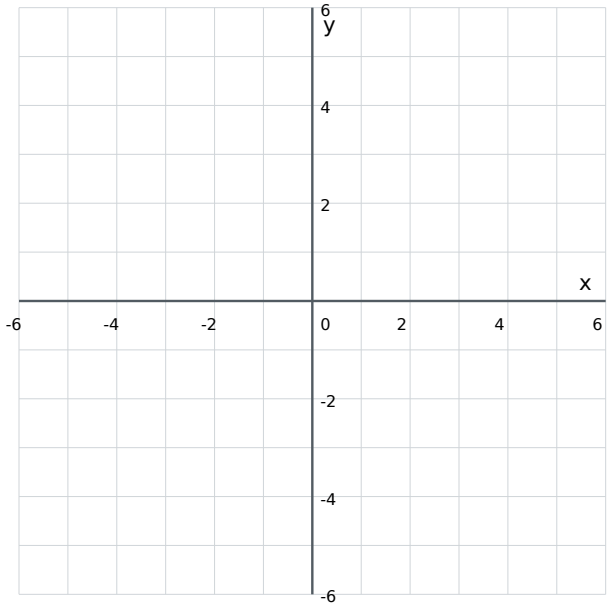
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



Unit 5 Review | 1

Simplify or solve. State all original restrictions and show work; check equation solutions.

1. $\dfrac{x^2-36}{x^2+11x+30}$

Answer / restrictions: _____

2. $\dfrac{x^2-4}{x^2+5x+6} \cdot \dfrac{x+3}{x-2}$

Answer / restrictions: _____

3. $\dfrac{x^2-25}{x+1} \div \dfrac{x-5}{x+1}$

Answer / restrictions: _____

4. $\dfrac{3}{x} - \dfrac{1}{x+2}$

Answer / restrictions: _____

Unit 5 Review | 2

Simplify or solve. State all original restrictions and show work; check equation solutions.

5. $\dfrac{1}{x^2-9}+\dfrac{2}{x+3}$

Answer / restrictions: _____

6. $\dfrac{\frac{1}{x}+\frac{1}{6}}{\frac{1}{x}-\frac{1}{6}}$

Answer / restrictions: _____

7. $\dfrac{5}{x-1}=\dfrac{2}{x+2}$

Answer / restrictions: _____

8. $\dfrac{x+1}{x-4}=\dfrac{5}{x-4}$

Answer / restrictions: _____

Unit 5 Review | Graphing

State domain, asymptotes and hole(s), if any. Sketch the function and label its features.

PROBLEM 9

$$y = \frac{-3}{x+2} + 1$$

Domain:

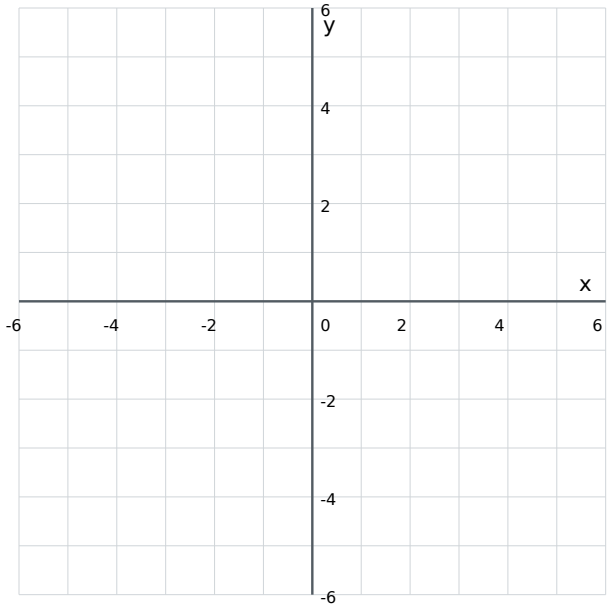
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



PROBLEM 10

$$y = \frac{x^2 - 9}{x^2 - x - 6}$$

Domain:

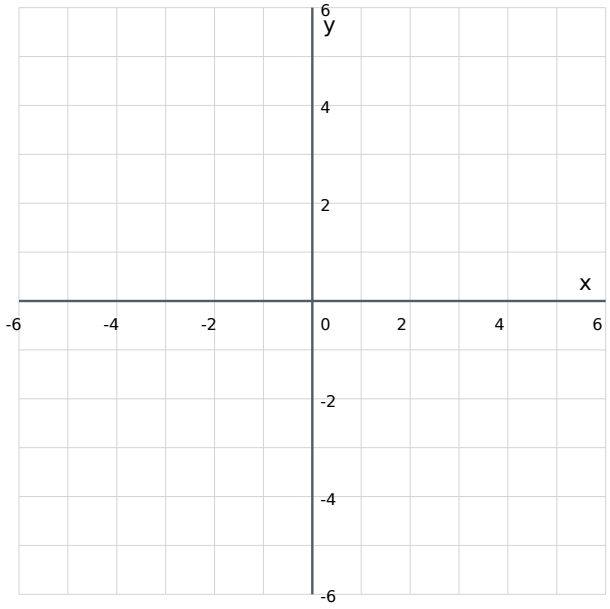
VA:

HA:

Hole(s):

Points (x, y):

Show factoring or describe transformations:



Waterworks Engineering

Teams of 2–3 • Use the existing virtual lab on the Unit 5 website. Record evidence here and export your final online report.

Group members: _____

STAGE 1 / EXPLORE & COLLECT EVIDENCE

Run six trials. Predict first, change one variable at a time for several trials, and include at least one obstruction trial. Use the displayed observed time; write “will not fill” for zero flow.

Trial	V (L)	A (L/min)	B (L/min)	Block %	Pred. min	Obs. min	Error %
1							
2							
3							
4							
5							
6							

Pattern observed (2–3 sentences):

STAGE 2 / BUILD & INTERPRET A RATIONAL MODEL

Set V = 120 L, A = 4 L/min, obstruction = 0%. Let x be Pump B flow. Test five nonnegative x-values. Record and plot them in the simulator.

x (L/min)					
T (min)					

Model: $T(x) = \frac{120}{4 + x}$

Algebraic domain: _____ Physical domain: _____

VA: _____ HA: _____ Which asymptote is physically relevant? _____

Waterworks | Solve & Troubleshoot

Continue in the online simulator. Include equations, units, and evidence from your tests.

STAGE 3 / EMERGENCY UPGRADE

A 120 L tank has Pump A = 4 L/min, Pump B = x L/min and 20% obstruction. It must fill in 15 minutes or less. Find the minimum Pump B flow, then verify it in the simulator.

$$T(x) = \frac{120}{0.8(4+x)} = \frac{150}{x+4}$$

Boundary equation and algebraic steps:

Minimum rate: _____ L/min Restrictions: _____

Substitution check / observed time: _____

STAGE 4 / TROUBLESHOOT A FAILURE

Set V = 120 L, A = 4 L/min, B = 0. Test obstruction at 0%, 50%, 90% and 100%. Use the model to predict each fill time.

0%	50%	90%	100%

Explain what happens as obstruction approaches 100% and why 100% gives no finite fill time.

Separate diagnostic function (not a hydraulic model):

$$D(x) = \frac{x-3}{(x-2)(x-3)}$$

Simplified: _____

Restrictions: _____

Hole: _____ VA: _____ Why is the canceled input still excluded? _____

Waterworks | Final Design

Stage 5 • Select equipment from the online catalog. Compare your design with another option and validate it.

DESIGN REQUIREMENTS

Store at least 120 L, fill within 18 minutes at 20% obstruction, and cost no more than 110 credits. Select a tank and pump configuration from the simulator catalog.

EQUIPMENT CATALOG / Credits

Tank: 120 L (25) | 150 L (36) | 180 L (48)

Pump A or B: 3 (15), 4 (20), 5 (26), 6 (33), 8 (45) L/min; Pump B may be 0 (0).

Tank (L)	Pump A (L/min)	Pump B (L/min)	Cost (credits)	Time (min)

Fill-time equation and cost calculations:

Compare with one alternative. Why is your design better for this brief?

Individual exit response: What does a domain restriction mean in a real water system?

FINAL REPORT / 40 POINTS

Data (6) • Rational model (8) • Algebraic reasoning (8) • Graph interpretation (6) • Engineering design (8) • Communication (4)

Generate the final report in the simulator and export or print it. Submit this packet and the report as directed.