WELCH TEACHING & COACHING

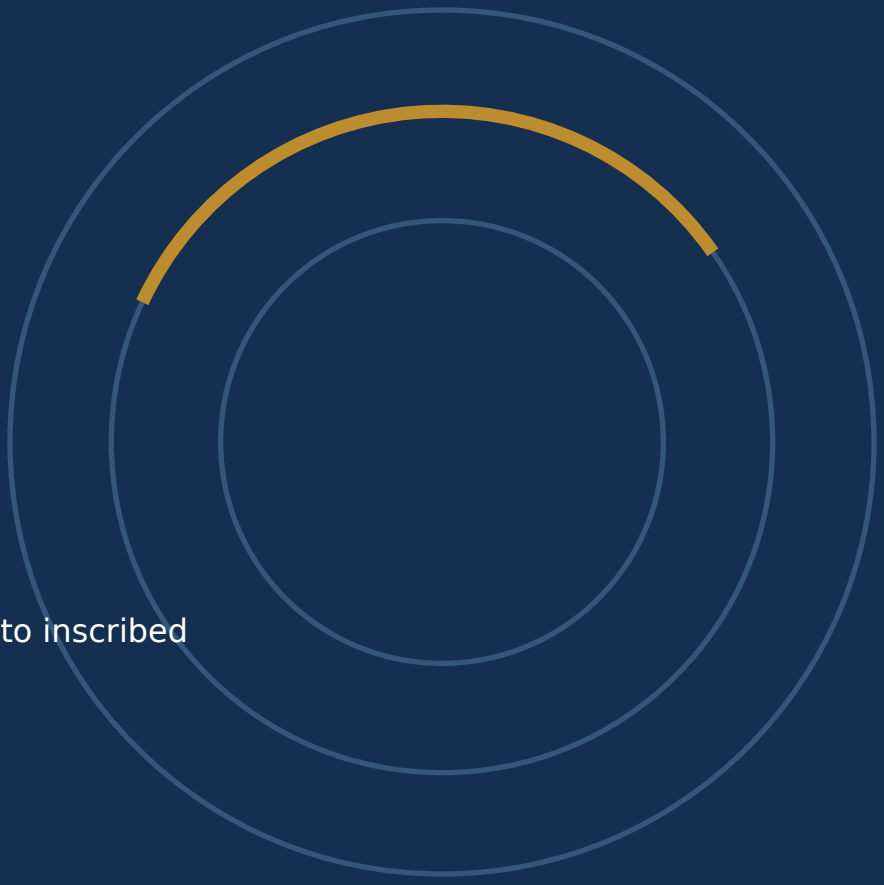
GEOMETRY

UNIT 5

CIRCLES

From chord distances and arc length to inscribed angles, tangents, secants, and cyclic quadrilaterals.

STUDENT WORKBOOK



Name _____

Class / Period _____

Date _____

UNIT PATH

5.1 Chords & Distance

5.2 Arcs & Sectors

5.3 Inscribed Angles

5.4 Tangents & Secants

5.5 Cyclic Quadrilaterals

How to use this workbook

Keep your notes and math work. Label diagrams before choosing formulas.

EVERY CIRCLE PROBLEM: FIVE STEPS

1 Identify	2 Mark	3 Select	4 Solve	5 Check
vertex, center, or tangent	radius, chord, arcs	appropriate theorem	write equation first	angle/length reasonable

REFERENCE — EQUATIONS TO EARN THROUGH THE UNIT

Chord–radius triangle	$a^2 + d^2 = r^2$ (a = half-chord)
Arc length (degrees)	$s = (\theta / 360°)(2\pi r)$
Sector area	$A = (\theta / 360°)(\pi r^2)$
Radians	$s = r\theta$ (θ in radians)
Inscribed angle	angle = ½(intercepted arc)
Exterior secant / tangent angle	angle = ½(far arc − near arc)
Cyclic quadrilateral	opposite angles sum to 180°

WORKBOOK CONVENTION

Each lesson includes STRIDES notes, three guided examples, six independent problems, and a two-question skills check. Use a ruler for chord markings and show equations before numerical answers.

5.1 STRIDES Notes

Circle Foundations: Chords, Radii, and Distance

S SUBJECT / TARGET

Use perpendicular radii and congruent-chord properties to form and solve right triangles.

RETRIEVAL — FROM PRIOR UNITS

1. Find the missing leg of a right triangle with hypotenuse 13 and one leg 5.

2. A 22-unit segment is bisected. Find each half.

D DEFINITIONS — ADD YOUR OWN

Circle:

Center:

Radius:

Diameter:

Chord:

Perpendicular bisector:

Congruent chords:

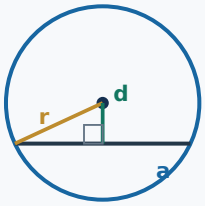
R RULES — NAME, DIAGRAM, AND EXPLAIN

Chord bisector:

Congruent chords:

Right triangle relation:

I ILLUSTRATION — LABEL WHAT YOU KNOW



E EXAMPLE / S STEPS

A circle has radius 10 and a chord 6 units from the center. Find the full chord length.

5.1 Guided Examples

Write the theorem first, then the equation, then the calculation.

EXAMPLE 1 DIRECT THEOREM

A circle has radius 10 and a chord 6 units from the center. Find the full chord length.

Relationship / equation: $(\text{half-chord})^2 + d^2 = r^2$

EXAMPLE 2 MULTI-STEP DIAGRAM

A circle has radius 13. A chord is 5 units from its center. Find the chord length.

Theorem and equation:

EXAMPLE 3 ALGEBRAIC COMPARISON

Two congruent chords are $3x + 1$ and $5x - 7$ units from the center. Find x.

Theorem and equation:

5.1 Independent Practice

Name the theorem. Draw, mark or annotate a diagram when helpful.

1. A radius is 10 and the perpendicular center-to-chord distance is 8. Find the full chord.

2. A radius is 17 and the perpendicular center-to-chord distance is 8. Find the full chord.

3. A chord is 24 units long in a circle of radius 13. Find its distance from the center.

4. Two congruent chords are $2x + 3$ and $5x - 9$ units from the center. Find x.

5. A circle has radius 15 and a chord of length 18. Find the distance from its center to the chord.

6. A park has circular radius 10 m. A straight bridge lies 6 m from the center. How long is the bridge?

5.1 Skills Check

Two questions • show the relationship and supporting work.

1 / 5 POINTS

A circle has radius 13 and a chord 12 units from the center. Find its full length.

Theorem or rule: _____

Work:

2 / 5 POINTS

Equal chords are $4x - 3$ and $2x + 7$ units from the center. Find x.

Theorem or rule: _____

Work:

REFLECT AND TRANSFER

What visual clue told you which rule to use? Write a sentence, not just an equation.

5.2 STRIDES Notes

Central Angles, Arc Length, and Sector Area

S SUBJECT / TARGET

Use the fraction of a circle determined by a central angle to calculate arc length and sector area, including reverse problems.

RETRIEVAL — FROM PRIOR UNITS

1. Find the circumference of a circle with radius 5.

2. Convert 60° to radians.

D DEFINITIONS — ADD YOUR OWN

Central angle:

Minor arc:

Major arc:

Arc length:

Sector:

Radian:

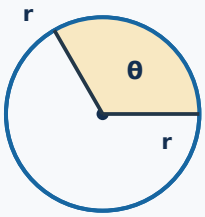
R RULES — NAME, DIAGRAM, AND EXPLAIN

Central angle:

Arc length:

Sector area:

I ILLUSTRATION — LABEL WHAT YOU KNOW



E EXAMPLE / S STEPS

A circle has radius 6 and a 60° central angle. Find the corresponding arc length.

5.2 Guided Examples

Write the theorem first, then the equation, then the calculation.

EXAMPLE 1 ARC LENGTH

A circle has radius 6 and a 60° central angle. Find the corresponding arc length.

Relationship / equation: part / whole = θ / 360°

EXAMPLE 2 SECTOR AREA

A circle has radius 8 and a 90° central angle. Find the sector area.

Theorem and equation:

EXAMPLE 3 WORK BACKWARD

An arc has length 6π in a circle with radius 9. Find its central angle.

Theorem and equation:

5.2 Independent Practice

Name the theorem. Draw, mark or annotate a diagram when helpful.

1. Find the arc length for radius 12 and central angle 90°.

2. Find the arc length for radius 6 and central angle 150°.

3. Find the sector area for radius 10 and central angle 72°.

4. A sector of radius 8 has area 16π. Find its central angle.

5. A circle has radius 15 and an arc length of 10π. Find the corresponding central angle.

6. Convert 210° to radians and find arc length if the radius is 12.

5.2 Skills Check

Two questions • show the relationship and supporting work.

1 / 5 POINTS

A central angle is 135° in a circle of radius 4. Find the arc length.

Theorem or rule:

Work:

2 / 5 POINTS

An arc has length 12π in a circle with radius 18. Find its central angle.

Theorem or rule:

Work:

REFLECT AND TRANSFER

What visual clue told you which rule to use? Write a sentence, not just an equation.

5.3 STRIDES Notes

Inscribed Angles and Intercepted Arcs

S SUBJECT / TARGET

Find intercepted arcs and inscribed angles; translate shared-arc relationships into equations.

RETRIEVAL — FROM PRIOR UNITS

1. Find the supplement of 112°.

2. Solve $3x - 5 = 64$.

D DEFINITIONS — ADD YOUR OWN

Inscribed angle:

Intercepted arc:

Central angle:

Semicircle:

Diameter:

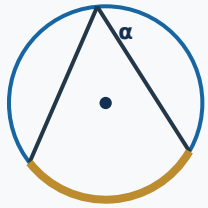
R RULES — NAME, DIAGRAM, AND EXPLAIN

Inscribed angle:

Shared intercepted arc:

Semicircle theorem:

I ILLUSTRATION — LABEL WHAT YOU KNOW



E EXAMPLE / S STEPS

An inscribed angle intercepts an arc of 82°. Find the angle.

5.3 Guided Examples

Write the theorem first, then the equation, then the calculation.

EXAMPLE 1 ONE INTERCEPTED ARC

An inscribed angle intercepts an arc of 82°. Find the angle.

Relationship / equation: angle = intercepted arc / 2

EXAMPLE 2 SAME INTERCEPTED ARC

Two inscribed angles intercept the same arc and measure 3x + 7 and 5x − 19 degrees. Find x.

Theorem and equation:

EXAMPLE 3 COMPARE CENTRAL AND INSCRIBED

A central angle measuring 146° and an inscribed angle intercept the same arc. Find the inscribed angle.

Theorem and equation:

5.3 Independent Practice

Name the theorem. Draw, mark or annotate a diagram when helpful.

1. An inscribed angle intercepts a 124° arc. Find the angle.

2. An inscribed angle is 34°. Find its intercepted arc.

3. A central angle is 156°. Find an inscribed angle intercepting the same arc.

4. Two inscribed angles intercept the same arc: $4x + 3$ and $7x - 24$ degrees. Find x.

5. A triangle is inscribed in a semicircle with the diameter as one side. Find its angle opposite that diameter.

6. An inscribed angle is $(3x - 1)°$ and its intercepted arc is $(4x + 18)°$. Find x and the angle.

5.3 Skills Check

Two questions • show the relationship and supporting work.

1 / 5 POINTS

An inscribed angle intercepts an arc of 146°. Find the angle.

Theorem or rule: _____

Work:

2 / 5 POINTS

An inscribed angle is (5x + 6)° and its intercepted arc is (12x − 4)°. Find x.

Theorem or rule: _____

Work:

REFLECT AND TRANSFER

What visual clue told you which rule to use? Write a sentence, not just an equation.

5.4 STRIDES Notes

Tangents, Secants, and Exterior Angles

S SUBJECT / TARGET

Find tangent–chord and exterior angles by identifying intercepted arcs and using half-sum/half-difference relationships.

RETRIEVAL — FROM PRIOR UNITS

1. A tangent touches a circle at T. Find the angle between tangent and radius OT.

2. Find 250° − 70°.

D DEFINITIONS — ADD YOUR OWN

Tangent:

Point of tangency:

Secant:

Exterior angle:

Tangent-chord angle:

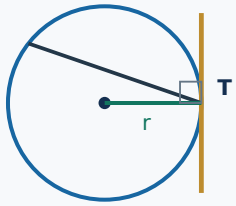
R RULES — NAME, DIAGRAM, AND EXPLAIN

Radius-tangent:

Tangent-chord:

Exterior arc difference:

I ILLUSTRATION — LABEL WHAT YOU KNOW



E EXAMPLE / S STEPS

A tangent and a chord form an angle intercepting a 130° arc. Find the angle.

5.4 Guided Examples

Write the theorem first, then the equation, then the calculation.

EXAMPLE 1 TANGENT AND CHORD

A tangent and a chord form an angle intercepting a 130° arc. Find the angle.

Relationship / equation: tangent–chord angle = arc / 2

EXAMPLE 2 TWO SECANTS OUTSIDE

Two secants intersect outside a circle. The far arc is 230° and near arc is 70°. Find the angle.

Theorem and equation:

EXAMPLE 3 TANGENT–SECANT OUTSIDE

A tangent and secant form an exterior angle. The far arc is 270° and near arc is 30°. Find the angle.

Theorem and equation:

5.4 Visual Guide: Locate the Vertex

Compare the three diagrams before choosing an angle theorem.

1. INSCRIBED ANGLE

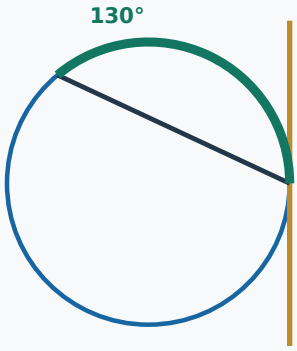


100° arc

Vertex on the circle • intercepted arc = 100°

Correct relationship:

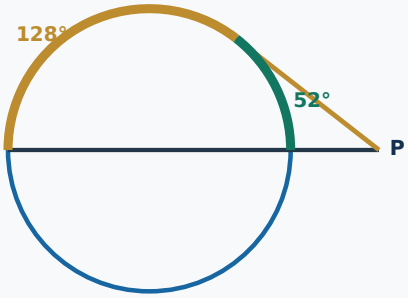
2. TANGENT–CHORD ANGLE



Vertex on the circle at the point of tangency • intercepted arc = 130°

Correct relationship:

3. TANGENT–SECANT EXTERIOR ANGLE



Vertex outside the circle • intercepted arcs = 128° and 52°

Correct relationship:

KEY DISTINCTION

A vertex on the circle uses half its intercepted arc. A vertex outside uses half the difference of the intercepted arcs. Identify the vertex before using a formula.

5.4 Independent Practice

Name the theorem. Draw, mark or annotate a diagram when helpful.

1. A tangent–chord angle intercepts an arc of 148°. Find the angle.

2. A tangent–chord angle measures 37°. Find the intercepted arc.

3. Two secants from an exterior point intercept arcs of 220° and 80°. Find the exterior angle.

4. A tangent and secant form an exterior angle with far/near arcs 250° and 50°. Find the angle.

5. The exterior angle measures 48°. Its larger intercepted arc is 164°. Find its smaller intercepted arc.

6. An exterior angle is (3x + 10)°, the larger intercepted arc is (8x + 20)°, and the smaller is 40°. Find x.

5.4 Skills Check

Two questions • show the relationship and supporting work.

1 / 5 POINTS

Two exterior secants intercept arcs 210° and 50°. Find the exterior angle.

Theorem or rule: _____

Work:

2 / 5 POINTS

An exterior angle is 56° and its larger intercepted arc is 188°. Find its smaller arc.

Theorem or rule: _____

Work:

REFLECT AND TRANSFER

What visual clue told you which rule to use? Write a sentence, not just an equation.

5.5 STRIDES Notes

Cyclic Quadrilaterals and Integrated Circle Problems

S SUBJECT / TARGET

Combine circle theorems and algebra to solve multi-unknown problems without being told which theorem to use.

RETRIEVAL — FROM PRIOR UNITS

1. Find the supplement of 78°.

2. Solve 2x + (3x + 5) = 180.

D DEFINITIONS — ADD YOUR OWN

Cyclic quadrilateral:

Opposite angles:

Supplementary angles:

Mixed diagram:

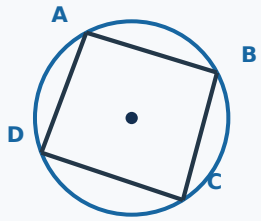
R RULES — NAME, DIAGRAM, AND EXPLAIN

Opposite angles:

Diagram-first method:

Independent equations:

I ILLUSTRATION — LABEL WHAT YOU KNOW



E EXAMPLE / S STEPS

Opposite angles in a cyclic quadrilateral measure 2x° and (3x + 10)°. Find x.

5.5 Guided Examples

Write the theorem first, then the equation, then the calculation.

EXAMPLE 1 ONE PAIR OF OPPOSITES

Opposite angles in a cyclic quadrilateral measure 2x° and (3x + 10)°. Find x.

Relationship / equation: opposite angles sum to 180°

EXAMPLE 2 TWO UNKNOWNS

Opposite pairs measure 2x° and (2x + 4)°, and y° and (3y + 8)°. Find x and y.

Theorem and equation:

EXAMPLE 3 MIXED RELATIONSHIPS

An inscribed angle intercepts a 124° arc and is a vertex of a cyclic quadrilateral. Find that vertex angle and the opposite angle.

Theorem and equation:

5.5 Independent Practice

Name the theorem. Draw, mark or annotate a diagram when helpful.

1. One angle of a cyclic quadrilateral is 78°. Find its opposite angle.

2. Opposite angles of a cyclic quadrilateral measure 4x° and (2x + 30)°. Find x.

3. Opposite angles are (5y + 10)° and (3y + 2)°. Find y.

4. Opposite angles are (2x + 8)° and (4x − 20)°. Find x and both angles.

5. In cyclic ABCD, ∠A = x + 25, ∠C = 3x − 5, ∠B = 2y + 8, and ∠D = 4y − 2. Find x and y.

6. An inscribed angle in a cyclic quadrilateral intercepts a 124° arc. Find the opposite vertex angle.

5.5 Skills Check

Two questions • show the relationship and supporting work.

1 / 5 POINTS

Opposite angles are $(2x + 14)°$ and $(4x - 2)°$. Find x and the angles.

Theorem or rule: _____

Work:

2 / 5 POINTS

Opposite pairs are $(3x + 2)°$ and $(2x + 18)°$, and $(4y + 6)°$ and $(2y + 24)°$. Find x and y.

Theorem or rule: _____

Work:

REFLECT AND TRANSFER

What visual clue told you which rule to use? Write a sentence, not just an equation.

Cumulative Review • Part 1

Mix theorems: the diagram does not tell you which formula to use.

1. A circle has radius 15 and a chord 18 long. Find the perpendicular center-to-chord distance.

2. Congruent chords lie at distances $(3x - 2)$ and $(x + 10)$. Find x.

3. Find the arc length of a 120° arc in a circle of radius 9.

4. A 72° sector has area 45π. Find the radius.

5. An inscribed angle intercepts a 118° arc. Find the angle.

Cumulative Review • Part 2

Mix theorems: the diagram does not tell you which formula to use.

6. Angles $(4x + 1)°$ and $(6x - 17)°$ intercept the same arc. Find x.

7. A tangent–chord angle intercepts a 154° arc. Find the angle.

8. Two exterior secants intercept arcs of 248° and 64°. Find their exterior angle.

9. Opposite cyclic angles measure $(2x + 20)°$ and $(4x - 8)°$. Find x.

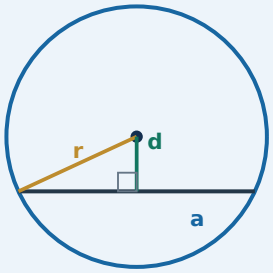
10. An inscribed angle intercepts a 92° arc. Find its opposite angle in a cyclic quadrilateral.

Performance Task • Circular Park Design

Use a labeled model and justify each geometric decision.

DESIGN BRIEF

The park is modeled by a circle of radius 13 m. A straight bridge crosses a chord 5 m from the center. The landscape team also wants a 90° arc and garden sector.



1. Bridge across the pond

Design a pond of radius 13 m. A straight pedestrian bridge runs 5 m from the center. Find bridge length.

2. Curved walking path

Using radius 13 m, find the length of a 90° shoreline arc.

3. Garden sector

Using radius 13 m, calculate the area of a 90° sector.

Performance Task • Circular Park Design

Use a labeled model and justify each geometric decision.

DESIGN BRIEF — CONTINUED

Use circle-angle relationships for a tangent path, two secant sightlines, and four stations placed on the same circular boundary.

4. Tangent pathway

A tangent path and fence chord intercept a 100° arc. Find the angle.

5. Observation point

Two secants from an exterior observation point intercept arcs of 240° and 60°. Find the viewing angle.

6. Four stations

Opposite angles in a cyclic quadrilateral measure $(2x + 10)°$ and $(3x - 5)°$. Find x and both angles.

FINAL JUSTIFICATION

Explain how at least three different circle theorems make your park calculations reliable.

Mastery Checklist and Reflection

Circle geometry becomes easier when you identify the figure before calculating.

- [ ] I can identify radius, chord, center, tangent, and secant.
- [ ] I can use a perpendicular from a center to bisect a chord.
- [ ] I can find arc length and sector area using a central angle.
- [ ] I can work backward from a length or area to an angle.
- [ ] I can identify the intercepted arc of an inscribed angle.
- [ ] I can use the tangent–chord angle relationship.
- [ ] I can use the exterior secant/tangent angle formula.
- [ ] I can solve opposite-angle equations in a cyclic quadrilateral.
- [ ] I can combine two or more circle theorems in one diagram.

THE NEXT STEP

Which two ideas should you revisit before the unit assessment? Describe one kind of diagram that still confuses you.