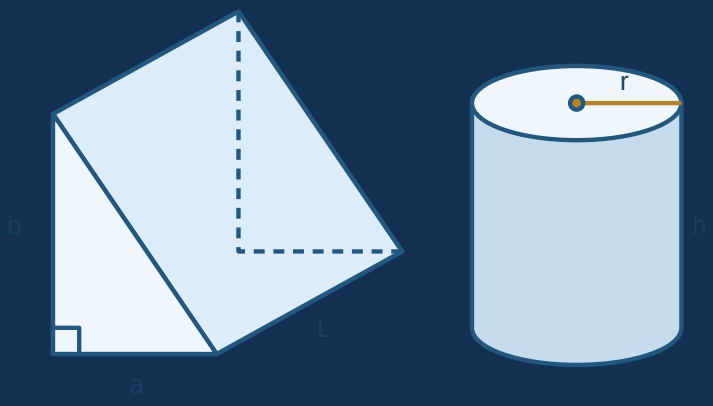
WELCH TEACHING & COACHING

GEOMETRY

UNIT 6

SOLID GEOMETRY

From faces and nets to volume, surface area, and engineering design.



Right triangular prism Cylinder

STUDENT WORKBOOK

Name _____

Class / Period _____

Date _____

6.1 Solid Geometry

6.2 Prisms & Cylinders

6.3 Pyramids & Cones

6.4 Spheres

6.5 Composite Solids

UNIT REFERENCE

How to use this workbook

Identify → mark dimensions → select the formula → solve → check units and reasonableness.

UNIT PATH

6.1 Solid Geometry	Pages 3–6
6.2 Prisms & Cylinders	Pages 7–10
6.3 Pyramids & Cones	Pages 11–14
6.4 Spheres	Pages 15–18
6.5 Composite Solids	Pages 19–22

EQUATION BANK

Euler's Theorem	$V - E + F = 2$ (convex polyhedra; V = vertices)
Prism / cylinder volume	$V = Bh$
Right prism / cylinder area	$SA = 2B + Ph$; cylinder: $2\pi r^2 + 2\pi rh$
Pyramid / cone volume	$V = Bh/3$
Regular pyramid / cone area	$SA = B + P\ell/2$; cone: $\pi r^2 + \pi r\ell$
Slant height	Cone: $\ell^2 = h^2 + r^2$; square pyramid: $\ell^2 = h^2 + (s/2)^2$
Sphere	$SA = 4\pi r^2$; $V = (4/3)\pi r^3$
Composite solids	Add or subtract volumes; count exposed faces only.
Supporting formulas	$A = \frac{1}{2}bh$; $A = \pi r^2$; $C = 2\pi r$; $a^2 + b^2 = c^2$

B = base area; P = base perimeter; h = perpendicular height; ℓ = slant height; r = radius; s = side length. Area: square units. Volume: cubic units. Keep π exact unless rounding is requested.

6.1 STRIDES NOTES

Introduction to Solid Geometry

Classify solids, count faces, edges and vertices, and connect solids to their nets.

S SUBJECT / TARGET

RETRIEVAL — FROM PRIOR UNITS

How many sides does a hexagon have?

Find the area of a 5 cm by 3 cm rectangle.

D DEFINITIONS — ADD YOUR OWN

Polyhedron:

Face:

Edge:

Vertex:

Base:

Net:

R RULES — NAME, DIAGRAM, AND EXPLAIN

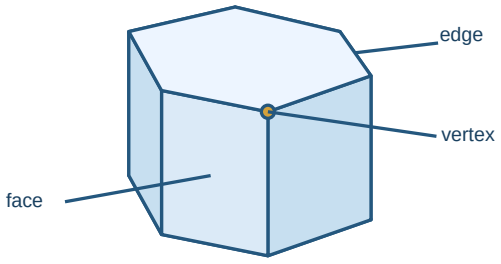
Euler’s Theorem

Prism structure

Pyramid structure

Nets

I ILLUSTRATION / E EXAMPLE / S STEPS



Hexagonal prism

Count faces, edges and vertices of a pentagonal prism.

6.1 GUIDED EXAMPLES

Solid Geometry

Write the formula, label the dimensions, substitute, solve and include units.

EXAMPLE 1

Count faces, edges and vertices of a pentagonal prism.

Count bases, side faces, and connecting edges.

Equation / work:

Answer: _____

EXAMPLE 2

A convex polyhedron has 14 faces and 24 vertices. Find the number of edges.

Start with $V - E + F = 2$.

Equation / work:

Answer: _____

EXAMPLE 3

Sketch a net of a triangular prism with a 3-4-5 right-triangle base and length 6 cm.

Draw two triangles and three rectangles; label corresponding sides.

Equation / work:

Answer: _____

6.1 INDEPENDENT PRACTICE

Solid Geometry

Show all work. Draw or annotate a diagram when useful. Use exact π unless asked to round.

PROBLEM 1

Count faces, edges and vertices of a triangular prism.

Equation / work:

Answer: _____

PROBLEM 2

Count faces, edges and vertices of a square pyramid.

Equation / work:

Answer: _____

PROBLEM 3

A convex polyhedron has 20 vertices and 12 faces. Find its edges.

Equation / work:

Answer: _____

PROBLEM 4

A convex polyhedron has 18 edges and 8 faces. Find its vertices.

Equation / work:

Answer: _____

PROBLEM 5

Sketch and label a net for a rectangular prism with dimensions 2, 3 and 5 cm.

Equation / work:

Answer: _____

PROBLEM 6

Classify a cylinder, cone, cube and pentagonal pyramid as polyhedra or non-polyhedra; explain your reasoning.

Equation / work:

Answer: _____

6.1 SKILLS CHECK

Solid Geometry

Two questions • 5 points each • Complete independently. Show your equation and work.

Name: _____ Date: _____

QUESTION 1 / 5 POINTS

Count faces, edges and vertices of a heptagonal prism. Verify Euler’s Theorem.

Equation / work:

Answer: _____

QUESTION 2 / 5 POINTS

A convex polyhedron has 15 edges and 9 vertices. Find its faces. Show the equation.

Equation / work:

Answer: _____

REFLECT AND TRANSFER

How can Euler’s Theorem help you catch a counting error?

6.2 STRIDES NOTES

Prisms and Cylinders

Use base area and perimeter to find volume, lateral area, total surface area and missing dimensions.

S SUBJECT / TARGET

RETRIEVAL — FROM PRIOR UNITS

Find the area of a triangle with base 8 cm and altitude 5 cm.

Find the area and circumference of a circle with diameter 10 cm.

D DEFINITIONS — ADD YOUR OWN

Volume:

Surface area:

Lateral area:

Cross section:

Perpendicular height:

Circumference:

R RULES — NAME, DIAGRAM, AND EXPLAIN

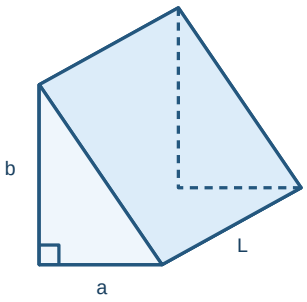
Prism and cylinder volume

Right prism surface area

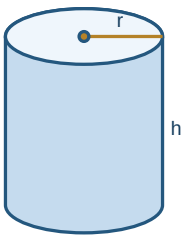
Cylinder surface area

Missing dimensions

I ILLUSTRATION / E EXAMPLE / S STEPS



Right triangular prism



Cylinder

A rectangular prism measures 4, 6 and 8 cm. Find volume and surface area.

6.2 GUIDED EXAMPLES

Prisms & Cylinders

Write the formula, label the dimensions, substitute, solve and include units.

EXAMPLE 1

A rectangular prism measures 4, 6 and 8 cm. Find volume and surface area.

Find base area and perimeter, then use Bh and 2B + Ph.

Equation / work:

Answer: _____

EXAMPLE 2

A right triangular prism has 9-12-15 cm bases and length 5 cm. Find volume and surface area.

B = ½bh; P = sum of triangle sides.

Equation / work:

Answer: _____

EXAMPLE 3

A cylinder has diameter 12 cm and height 7 cm. Find volume and surface area.

Find radius first; include both circular bases.

Equation / work:

Answer: _____

6.2 INDEPENDENT PRACTICE

Prisms & Cylinders

Show all work. Draw or annotate a diagram when useful. Use exact π unless asked to round.

PROBLEM 1

Find V and SA of a 5 × 7 × 9 in. rectangular prism.

Equation / work:

Answer: _____

PROBLEM 2

Find V and SA of a right triangular prism with legs 8 and 15 cm and length 4 cm.

Equation / work:

Answer: _____

PROBLEM 3

A cylinder has radius 3 m and height 12 m. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 4

A cylinder has diameter 16 cm and height 5 cm. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 5

A rectangular prism has volume 420 ft³, length 12 ft and width 5 ft. Find height.

Equation / work:

Answer: _____

PROBLEM 6

A cylinder has volume 392π cm³ and height 8 cm. Find radius.

Equation / work:

Answer: _____

6.2 SKILLS CHECK

Prisms & Cylinders

Two questions • 5 points each • Complete independently. Show your equation and work.

Name: _____ Date: _____

QUESTION 1 / 5 POINTS

A right triangular prism has legs 3 cm and 4 cm and length 10 cm. Find V and total SA.

Equation / work:

Answer: _____

QUESTION 2 / 5 POINTS

A closed cylinder has diameter 10 cm and height 9 cm. Find V and total SA.

Equation / work:

Answer: _____

REFLECT AND TRANSFER

Which parts of a cylinder’s net explain the surface area formula?

6.3 STRIDES NOTES

Pyramids and Cones

Distinguish slant height from vertical height; calculate volume and surface area of pyramids and cones.

S SUBJECT / TARGET

RETRIEVAL — FROM PRIOR UNITS

Find the hypotenuse when legs are 6 and 8.

Find the area of a triangle with base 12 and altitude 5.

D DEFINITIONS — ADD YOUR OWN

Apex:

Slant height:

Vertical height:

Regular pyramid:

Lateral face:

R RULES — NAME, DIAGRAM, AND EXPLAIN

Pyramid and cone volume

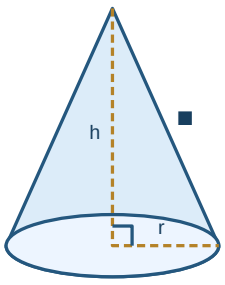
Regular pyramid surface area

Right cone surface area

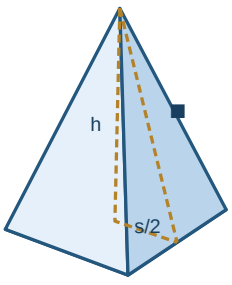
Pythagorean height relation

Rectangular pyramid

I ILLUSTRATION / E EXAMPLE / S STEPS



Cone



Right square pyramid

Find V of a rectangular pyramid with base 7 × 6 cm and height 9 cm.

6.3 GUIDED EXAMPLES

Pyramids & Cones

Write the formula, label the dimensions, substitute, solve and include units.

EXAMPLE 1

Find V of a rectangular pyramid with base 7 × 6 cm and height 9 cm.

Compute B; use one-third of Bh.

Equation / work:

Answer: _____

EXAMPLE 2

A right square pyramid has base side 8 cm and height 3 cm. Find V and SA.

Find ℓ using half the base side and vertical height.

Equation / work:

Answer: _____

EXAMPLE 3

A cone has radius 9 cm and slant height 15 cm. Find V and SA.

Find vertical height using the Pythagorean Theorem.

Equation / work:

Answer: _____

6.3 INDEPENDENT PRACTICE

Pyramids & Cones

Show all work. Draw or annotate a diagram when useful. Use exact π unless asked to round.

PROBLEM 1

Find V of a rectangular pyramid with base 12 × 7 m and height 5 m.

Equation / work:

Answer: _____

PROBLEM 2

A right square pyramid has s = 6 cm and h = 4 cm. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 3

A right square pyramid has s = 16 in. and slant height 17 in. Find vertical height and V.

Equation / work:

Answer: _____

PROBLEM 4

A cone has r = 6 cm and h = 8 cm. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 5

A cone has diameter 14 m and slant height 25 m. Find total SA.

Equation / work:

Answer: _____

PROBLEM 6

A right rectangular pyramid has base 8 × 6 cm and h = 4 cm. Find total SA (use two different slant heights).

Equation / work:

Answer: _____

6.3 SKILLS CHECK

Pyramids & Cones

Two questions • 5 points each • Complete independently. Show your equation and work.

Name: _____ Date: _____

QUESTION 1 / 5 POINTS

A right square pyramid has s = 18 cm and vertical height 12 cm. Find V and total SA.

Equation / work:

Answer: _____

QUESTION 2 / 5 POINTS

A cone has r = 7 cm and h = 24 cm. Find V and total SA.

Equation / work:

Answer: _____

REFLECT AND TRANSFER

Why is half the base side used to find the slant height of a square pyramid?

6.4 STRIDES NOTES

Spheres

Calculate surface area and volume of spheres and work backward to determine radius or diameter.

S SUBJECT / TARGET

RETRIEVAL — FROM PRIOR UNITS

Find the radius if diameter is 18 cm.

Evaluate $\sqrt{81}$ and $\sqrt[3]{125}$.

D DEFINITIONS — ADD YOUR OWN

Sphere:

Radius:

Diameter:

Great circle:

Cube root:

R RULES — NAME, DIAGRAM, AND EXPLAIN

Sphere surface area

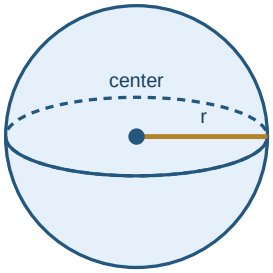
Sphere volume

Inverse sphere volume

Inverse sphere area

Scaling

I ILLUSTRATION / E EXAMPLE / S STEPS



Diameter = 2r

A sphere has diameter 16 cm. Find SA and V.

6.4 GUIDED EXAMPLES

Spheres

Write the formula, label the dimensions, substitute, solve and include units.

EXAMPLE 1

A sphere has diameter 16 cm. Find SA and V.

Use r = 8 cm.

Equation / work:

Answer: _____

EXAMPLE 2

A sphere has volume 256π/3 cm³. Find radius.

Isolate r^3 before taking a cube root.

Equation / work:

Answer: _____

EXAMPLE 3

A sphere has surface area 324π m². Find diameter.

Isolate r^2; find radius, then double.

Equation / work:

Answer: _____

6.4 INDEPENDENT PRACTICE

Spheres

Show all work. Draw or annotate a diagram when useful. Use exact π unless asked to round.

PROBLEM 1

Find SA and V of a sphere with radius 7 cm.

Equation / work:

Answer: _____

PROBLEM 2

Find SA and V of a sphere with diameter 10 in.

Equation / work:

Answer: _____

PROBLEM 3

A sphere has $V = 972\pi$ cm³. Find radius.

Equation / work:

Answer: _____

PROBLEM 4

A sphere has $V = 1436.76$ m³. Find radius to the nearest tenth.

Equation / work:

Answer: _____

PROBLEM 5

A sphere has $SA = 64\pi$ ft². Find its diameter.

Equation / work:

Answer: _____

PROBLEM 6

A sphere's radius changes from 4 cm to 8 cm. Find the ratios of new to original area and volume.

Equation / work:

Answer: _____

6.4 SKILLS CHECK

Spheres

Two questions • 5 points each • Complete independently. Show your equation and work.

Name: _____ Date: _____

QUESTION 1 / 5 POINTS

A sphere has diameter 14 cm. Find total SA and V in terms of π.

Equation / work:

Answer: _____

QUESTION 2 / 5 POINTS

A sphere has volume 500π/3 ft³. Set up and solve an equation for its radius.

Equation / work:

Answer: _____

REFLECT AND TRANSFER

Explain how you know whether to use a square root or cube root.

6.5 STRIDES NOTES

Composite Solids and Applications

Break complex solids into familiar pieces, calculate total volume and exposed area, and apply the results to engineering design.

S SUBJECT / TARGET

RETRIEVAL — FROM PRIOR UNITS

Find V of a 4 × 5 × 6 cm box.

Find the area of a circle with radius 3 cm.

D DEFINITIONS — ADD YOUR OWN

Composite solid:

Decomposition:

Exposed surface:

Capacity:

Material efficiency:

Percent error:

R RULES — NAME, DIAGRAM, AND EXPLAIN

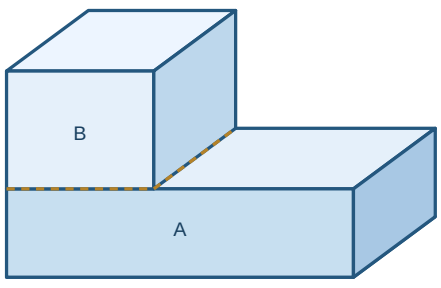
Composite volume

Composite surface area

Engineering constraints

Percent error

I ILLUSTRATION / E EXAMPLE / S STEPS



Decompose at the shared face

A step has lower box 9 × 4 × 3 cm and upper box 3 × 4 × 6 cm. Find V.

6.5 GUIDED EXAMPLES

Composite Solids

Write the formula, label the dimensions, substitute, solve and include units.

EXAMPLE 1

A step has lower box 9 × 4 × 3 cm and upper box 3 × 4 × 6 cm. Find V.

Compute each box separately.

Equation / work:

Answer: _____

EXAMPLE 2

A 9 × 8 × 5 cm block has a 3 × 2 × 5 cm tunnel removed. Find remaining V.

Subtract the hole volume.

Equation / work:

Answer: _____

EXAMPLE 3

A cylinder with r = 4 cm and h = 6 cm is joined to a cone with r = 4 cm and h = 3 cm. Find total V and exposed SA.

Only the bottom disk, cylinder wall and cone wall are exposed.

Equation / work:

Answer: _____

6.5 INDEPENDENT PRACTICE

Composite Solids

Show all work. Draw or annotate a diagram when useful. Use exact π unless asked to round.

PROBLEM 1

A lower box is 14 × 5 × 4 cm; an upper box is 6 × 5 × 3 cm. Find total V.

Equation / work:

Answer: _____

PROBLEM 2

A step is 12 cm long and 4 cm deep. The lower height is 3 cm; the leftmost 5 cm rises to total height 9 cm. Find V.

Equation / work:

Answer: _____

PROBLEM 3

A 12 × 9 × 7 cm block has a 4 × 3 × 7 cm tunnel removed. Find V.

Equation / work:

Answer: _____

PROBLEM 4

A cylinder r = 5 cm, h = 8 cm is topped by a cone r = 5 cm, h = 12 cm. Find total V.

Equation / work:

Answer: _____

PROBLEM 5

Two boxes 6 × 4 × 3 cm and 6 × 4 × 2 cm are joined along their 6 × 4 cm faces. Find exposed SA.

Equation / work:

Answer: _____

PROBLEM 6

A package is predicted to hold 260 cm^3; measured internal volume is 253 cm^3. Find percent error to the nearest tenth.

Equation / work:

Answer: _____

6.5 SKILLS CHECK

Composite Solids

Two questions • 5 points each • Complete independently. Show your equation and work.

Name: _____ Date: _____

QUESTION 1 / 5 POINTS

A stepped solid is 15 cm long and 6 cm deep. Its lower height is 4 cm; the leftmost 5 cm rises to total height 10 cm. Find total V. Show both pieces.

Equation / work:

Answer: _____

QUESTION 2 / 5 POINTS

A cylinder with r = 3 cm and h = 7 cm is topped by a cone with r = 3 cm and h = 4 cm. Find total V and exposed SA.

Equation / work:

Answer: _____

REFLECT AND TRANSFER

Why can two solids with the same volume require different amounts of material?

CUMULATIVE REVIEW

Mixed Solid Geometry — 1 of 2

Select the correct relationship independently. Show each equation and all supporting work.

PROBLEM 1

A pentagonal pyramid: find F, E, V and verify Euler.

Equation / work:

Answer: _____

PROBLEM 2

A rectangular prism measures 8 × 5 × 11 cm. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 3

A right triangular prism has legs 10 and 24 cm and length 7 cm. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 4

A cylinder has diameter 6 m and height 14 m. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 5

A right square pyramid has side 10 cm and height 12 cm. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 6

A cone has radius 12 in. and height 5 in. Find V and SA.

Equation / work:

Answer: _____

CUMULATIVE REVIEW

Mixed Solid Geometry — 2 of 2

Select the correct relationship independently. Show each equation and all supporting work.

PROBLEM 7

A sphere has diameter 20 cm. Find V and SA.

Equation / work:

Answer: _____

PROBLEM 8

A sphere has V = 2304π cm³. Find radius.

Equation / work:

Answer: _____

PROBLEM 9

A cylinder has V = 200π ft³ and radius 5 ft. Find height.

Equation / work:

Answer: _____

PROBLEM 10

A lower box measures 16 × 4 × 5 cm; an upper box is 6 × 4 × 7 cm. Find total V.

Equation / work:

Answer: _____

PROBLEM 11

A cylinder with r = 6 cm and h = 8 cm has a cone of same radius and height 8 cm on top. Find V and exposed SA.

Equation / work:

Answer: _____

PROBLEM 12

A package has predicted V = 264 cm³ and actual V = 258 cm³. Find percent error to the nearest tenth.

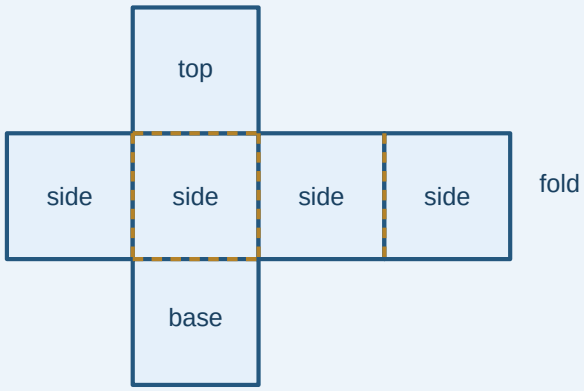
Equation / work:

Answer: _____

VISUAL REFERENCE

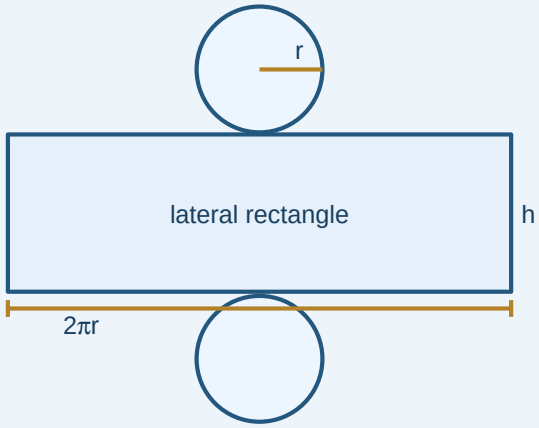
From a solid to its net

Use nets to identify every surface. A net is a flat pattern, not a volume diagram.



Cube net

Six congruent squares. Each square has area s^2. Total surface area = $6s^2$. The dashed gold lines indicate folds.



Cylinder net

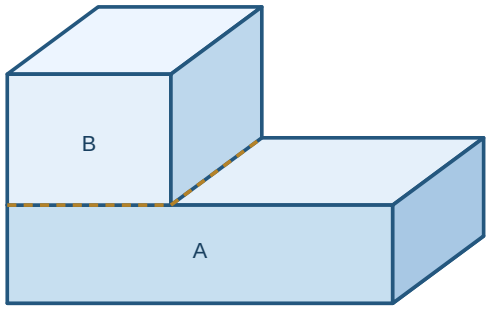
Two circles plus a rectangle. Each circle has area πr^2. The rectangle measures $2\pi r$ by h. Total surface area = $2\pi r^2 + 2\pi rh$.

Sketch a right triangular prism net. Label both triangles and all three rectangles.

PERFORMANCE TASK

Packaging Engineering Design Challenge

Five-block project • use the separate project packet and Engineering Studio.



Decompose at the shared face

Design a closed container that holds 250–275 cm³. Use no more than two sheets per prototype. A cube is reserved for the worked example; a sphere is for digital exploration only.

BLOCK 1 • Explore three competing designs in the studio.

BLOCK 2 • Complete hand calculations and a labeled net; get approval.

BLOCK 3 • Construct the first prototype and record materials.

BLOCK 4 • Measure, test, redesign and construct the second prototype.

BLOCK 5 • Retest, compare, present and complete your individual reflection.

Project resources: Unit6_Studio.html • Unit6_Student_Packet.pdf • Unit6_Cube_Example.pdf. Your teacher will provide the separate 100-point project packet.

Before the project begins: Which formulas or skills will you need to revisit?