Packaging Engineering Design Challenge

Five blocks • Design digitally, build physically, test, and improve

Team name: _____ **Class:** _____

Team members: _____

The engineering mission

You are a packaging engineer. Design a closed container that holds the required amount of a product while using as little material as practical. Your team will compare digital designs, create dimensioned plans, build two physical prototypes, test them, and explain the improvements using mathematics.

Requirement	Specification
Capacity	250–275 cm^3 of modeled internal volume; at least 250 mL of dry fill must fit and close.
Materials	At most two US Letter sheets per prototype (four total); scissors, ruler, pencil, tape; compass if needed.
Buildable shapes	Rectangular prism (not a cube), right triangular prism, cylinder, right square pyramid, or right cone. Sphere: digital comparison only.
Function	Closed package with a reusable opening; stable when filled; seams hold during testing.
Evidence	Three candidates, hand calculations, two construction plans, two prototypes, tests and final report.

Five-block roadmap

Block	Focus	Checkpoint
1	Explore & compare	Three distinct designs; choose one.
2	Model & plan	Hand calculations and dimensioned net; approval.
3	Build	Prototype 1; material log and build notes.
4	Test & redesign	Test 1, revise calculations, construct prototype 2.
5	Validate & present	Test 2, evidence-based report and reflection.

Team roles: Modeling lead, construction lead, testing/documentation lead. Rotate roles each block; every student completes the individual reflection.

Geometry & Engineering Toolkit

Keep this page open during planning, testing and redesign.

Vocabulary

Base: a face or region used to measure the solid. **Perpendicular height:** distance at a right angle to the base. **Slant height:** distance along a cone or pyramid face from base edge to apex. **Net:** flat arrangement of a solid's faces. **Prototype:** a testable model. **Constraint:** a requirement a design must satisfy.

Solid	Volume (cm^3)	Total surface area (cm^2)
Rectangular prism	$V = lwh$	$SA = 2(lw + lh + wh)$
Right triangular prism	$V = \frac{1}{2}abL$	$SA = ab + L(a + b + c)$; $c = \sqrt{(a^2 + b^2)}$
Cylinder	$V = \pi r^2 h$	$SA = 2\pi r^2 + 2\pi rh$
Right square pyramid	$V = a^2h / 3$	$SA = a^2 + 2as$; $s = \sqrt{(h^2 + (a/2)^2)}$
Right cone	$V = \pi r^2 h / 3$	$SA = \pi r^2 + \pi rs$; $s = \sqrt{(r^2 + h^2)}$
Sphere (digital only)	$V = 4\pi r^3 / 3$	$SA = 4\pi r^2$

Symbols: l = length; w = width; h = perpendicular height; r = radius; a, b = right-triangle legs (a = square base side for pyramid); c = hypotenuse; L = prism length; s = slant height. Use calculator π; round final values to 2 decimal places.

Engineering measurements

Material efficiency: $E = V / SA$ (cm^3/cm^2). Higher E means more modeled storage per unit of surface area. Compare designs at similar volumes.
Volume percent error: |measured-dimension V − planned V| / planned V × 100%.
Actual material efficiency: measured-dimension V / estimated cardstock area used.
Material use: panel area + cardstock tabs/overlaps. Do not count tape or reusable scraps as cardstock used. One Letter sheet = 21.59 × 27.94 cm ≈ 603.22 cm^2.

Worked example: cube (teacher model only)

A cube with edge 6.4 cm has $V = 6.4^3 = 262.14$ cm^3 and $SA = 6(6.4^2) = 245.76$ cm^2.
Its efficiency is 262.14 / 245.76 = 1.07 cm^3/cm^2. The model meets the capacity target.
See the separate Cube Worked Example for the net, testing and redesign.
Student designs may not be cubes, including rectangular prisms with all three dimensions equal.

Explore and Compare Designs

Studio sections 1–2 • Aim for 15 min explore, 35 min compare, 20 min decide.

Use the Shape Explorer to investigate at least three different shape families. Each model must have 250–275 cm³ capacity. Save your final versions in A, B and C and record them below. A sphere may be a digital candidate only. Cubes are reserved for the teacher example.

Record	Design A	Design B	Design C
Shape			
Dimensions (cm)			
Volume (cm³)			
Surface area (cm²)			
Efficiency (V/SA)			
Template sheets / build difficulty			
Practical advantage or concern			

Investigate one change

Choose one candidate. Change at least one dimension and keep the capacity within the target. Record the original and revised values.

Version	Changed dimension(s)	V (cm³)	SA (cm²)	Efficiency
Before				
After				

Which candidate will you build? Compare at least two designs numerically and explain one practical trade-off. Save your choice as Prototype 1 in the studio.

Prototype 1: Mathematical Model

Studio sections 2–3 • Complete by hand before cutting any cardstock.

Chosen shape: _____ Planned dimensions (cm): _____

1. Draw the 3D solid. Label every dimension, base, height and any slant height.

2. Find any missing dimensions (Pythagorean Theorem, radius, etc.). Show work.

3. Calculate volume. Write the formula, substitute dimensions, solve and label units.

4. Calculate total surface area. Include all faces or curved surfaces; show your work.

5. Calculate V/SA. State whether the capacity and efficiency support your selection.

Prototype 1: Construction Plan

Studio section 3 • Draw first; use the digital template to check your work.

Draw a complete dimensioned net or set of cut panels. Label solid cut lines, dashed fold lines, matching edges, and the opening/closure. For a cylinder, include two circles and a rectangle; for a cone, include the base circle and sector. Use tape at seams or add measured tabs.

Net / panel engineering drawing (grid for planning; not actual scale)

Planning check	Your result
Template sheet estimate (maximum 2) / planned actual sheets	
Panel area (cm²) + planned extra cardstock (cm²) = total planned material	
How will your package open and close? Which face is the lid?	

Before cutting: Check scale on the printed 5 cm line, confirm matching edges, and obtain teacher approval. If the studio finds no layout, propose a manual layout for approval.

Teacher approval: _____ Date: _____

Construct Prototype 1

Build full scale • Use at most two sheets for this version.

Build sequence

1. Transfer or print the full-scale templates. Verify the 5 cm calibration line. 2. Cut and fold carefully. 3. Join the seams with tape and make one face openable. 4. Record changes and preserve the original design plan. 5. Label your prototype and save all scraps for the material-use estimate.

Material log	Quantity / area
Number of sheets used (max 2)	
Total area of sheets used (count × 603.22 cm²)	
Estimated cut panel area (cm²)	
Extra cardstock tabs / overlap (cm²)	
Estimated actual cardstock used = panel area + extras (cm²)	
Estimated unused paper = sheet area − actual cardstock used (cm²)	

Estimate panel area from your actual cut dimensions or from the net if pieces match the plan. Measure extra tabs as rectangles or triangles. If a piece is discarded, record it as unused/waste; do not count the same area twice.

Construction sketch / photograph notes: mark seams, closure, and any changes.

What was the biggest challenge during construction? How did you solve it?

Prototype 1 complete: _____ Team roles rotated: _____

Test Prototype 1

Studio section 4 • Record results before choosing a redesign.

Use the same dry fill, measuring cup, surface and handling procedure for both prototypes. Pour gently without packing. Measure internal dimensions; use the appropriate solid formula. If the shape is imperfect, record the limitation rather than claiming exact volume.

Test	Procedure	Result / evidence
Internal dimensions	Record actual dimensions (cm); measure the radius or diameter where appropriate.	
Capacity: 250 mL	Fill with 250 mL of dry rice or equivalent; does it close without spilling?	
Capacity: 275 mL	Try 275 mL; note whether it fits. This is an approximate check, not a precise upper-volume test.	
Stability	Set filled package on a level table. Does it stand without support?	
Closure	Open and close five times. Does it still close securely?	
Seams / durability	Lift and set down filled container three times. Check for gaps or failure.	

Measured-dimension volume: formula, substitution, answer and percent error.

Measured-dimension surface area and actual material efficiency (V / cardstock used).

What did the physical tests reveal that the digital model did not?

Redesign: Prototype 2

Use test evidence to make one measurable, meaningful improvement.

Choose a change: reduce material, improve stability or closure, reduce waste, or make assembly easier. The second prototype still needs 250–275 cm^3 modeled capacity and no more than two sheets.

1. What problem did you identify? Cite at least one test result and explain your proposed change.

Design comparison	Prototype 1 (planned)	Prototype 2 (planned)
Shape and dimensions (cm)		
Volume (cm^3)		
Surface area (cm^2)		
Efficiency (V/SA)		
Predicted cardstock used (cm^2) / sheet count		

2. Recalculate Prototype 2 volume and surface area by hand. Show formula and substitution.

3. Predict how the change will affect performance. Use a numerical comparison (difference or percent change) when possible.

Save the revised model as Prototype 2 in the studio. Draw the revised construction plan on the next page.

Prototype 2: Build Plan

Revised net or cut panels • Build the improved design after approval.

Revised dimensioned net: show cutting/folding lines, matching edges, and closure.

Material record	Prototype 2
Sheets used (maximum 2)	
Cut panel area (cm²)	
Extra tabs / overlap (cm²)	
Actual cardstock used (cm²)	
Estimated unused paper (cm²)	

What changed during construction? Explain any difference between your revised plan and the finished prototype.

Teacher approval / build completion: _____

Test Prototype 2

Repeat the same tests; compare with your original prototype.

Use the same dry fill, measuring cup, surface and handling procedure for both prototypes. Pour gently without packing. Measure internal dimensions; use the appropriate solid formula. If the shape is imperfect, record the limitation rather than claiming exact volume.

Test	Procedure	Result / evidence
Internal dimensions	Record actual dimensions (cm); measure the radius or diameter where appropriate.	
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Measured-dimension volume: formula, substitution, answer and percent error.

Measured-dimension surface area and actual material efficiency (V / cardstock used).

What did the physical tests reveal that the digital model did not?

Final Engineering Report

Studio section 5 • Prepare a 4–5 minute team presentation.

Measure	Prototype 1	Prototype 2	Change / improvement
Measured volume (cm^3)			
Modeled surface area (cm^2)			
Actual cardstock used (cm^2)			
Actual efficiency (cm^3/cm^2)			
Volume error (%)			
Capacity / closure / stability			

1. Which prototype performed better overall? Support your claim with at least two numerical comparisons and one physical test result.

2. Explain a trade-off between mathematical efficiency and manufacturability. What would you change with more time?

Presentation checklist

☐ Show both prototypes. ☐ Explain your chosen shape and calculations. ☐ Describe the test result that drove the redesign. ☐ Compare final evidence. ☐ Every team member speaks. ☐ Print or save the studio summary and submit it with the packet.

Individual Reflection and Grading

Each student completes a separate reflection; copy this page if needed.

Name: _____ Team: _____

1. Describe your own contribution to the mathematics, construction, or testing.

2. What geometry concept did this project help you understand? Use a specific example.

3. If the company manufactured 1,000 packages, why would small material savings matter?

100-point scoring guide

Category	Full-credit evidence	Pts
Digital exploration	Three distinct viable models; accurate comparison and justified choice.	15
Mathematical model	Correct formulas, dimensions, units, calculations and efficiencies for both versions.	25
Drawings / nets	Complete, dimensioned plans; cut/fold lines, closure, sheet check.	15
Prototype 1	Full-scale, functional build; materials and construction documented.	10
Testing & analysis	Consistent tests for both versions; measured values, error and evidence.	10
Redesign	Specific evidence-driven change, recalculation and improved prototype.	15
Presentation / reflection	Clear comparison with evidence; each student participates and reflects.	10
TOTAL		100

Assessment is based on mathematical reasoning and engineering evidence, not on which team has the lowest surface area. A failed test can earn full analysis credit when documented and addressed.