Mathematical Foundations

One shared beginning • Two math pathways • Hands-on reasoning

START HERE • ALL COURSES

Name: _____ Class: _____ Date: _____

Your route: Everyone completes pages 1-4, 10-12, 13-14 and 17. Honors Physics / ECE1201 complete pages 5-6 and 15. AP Physics C/ECE1401 complete pages 7-9 and 16. Your teacher may assign additional pages for review.

How to work through the unit

- Read the lesson notes and demonstrations on /physics/unit0/.
- Predict before using any simulator; then test, collect evidence, and explain what changed.
- Write units with numerical results and identify the physical meaning of each graph or equation.
- Use STRIDES when taking notes; do not copy every sentence of the website.

STRIDES notes - a repeatable process

S / T	R / I	D / E / S
Subject + Topic	Rules + Illustrations	Definitions + Example + Steps

Vocabulary to know: measurement, dimension, vector, scalar, uncertainty, slope, model, component, derivative, integral.

What is physics modeling? Explain with one example.

Personal goal: Which math tool do you need to strengthen?

Online companion: welchteachingandcoaching.com/physics/unit0/ • Use the algebra or calculus tab as assigned.

Measurement, Units & Conversions

Shared lesson 0.1 • Record calculations and explain units

COMMON FOUNDATIONS • PAGE 2

Vocabulary: SI base unit, derived unit, dimension, precision, accuracy, conversion factor.

Rules to add: 1 km = 1000 m; 1 h = 3600 s; 1 cm = 10^{-2} m; units of area and volume convert with powers.

Worked setup - units act like algebra

72 km/h × (1000 m / 1 km) × (1 h / 3600 s) = 20 m/s

1. Convert 31.0 m/s to km/h. Show cancellation.

2. Convert 125 cm² to m². Explain why you square the conversion.

3. A density is 1.20 g/cm³. Convert it to kg/m³.

STRIDES: Sketch or explain a real-world quantity that needs a derived unit.

Check before continuing: a numerical conversion should change the number but not the measured physical quantity.

Dimensions, Error & Uncertainty

Shared lesson 0.2 • Evaluate what measurements can support

COMMON FOUNDATIONS • PAGE 3

Reference: length [L], mass [M], time [T]; velocity [L]/[T]; acceleration [L]/[T]^2.

4. Show that $F = mv^2/r$ has dimensions of force. Write every dimension.

5. Explain why $x = x_0 + v_0t + \frac{1}{2}at^2$ is dimensionally consistent.

Measurement station

A group measures a small object three times: 24.8 g, 25.1 g, 25.0 g. The balance reads to 0.1 g. For this exercise, use half the range as a measure of repeatability.

Trial 1	Trial 2	Trial 3	Mean	Half-range
24.8 g	25.1 g	25.0 g	_____	_____

6. Report the mean and half-range in the form value ± uncertainty, with units.

7. Estimate percent uncertainty, then name one possible systematic error.

> Teacher note for later labs: instrument resolution and repeatability are different sources of uncertainty. The half-range convention here is a simple estimate, not a universal propagation rule.

Graphs, Scalars & Vectors

Shared lesson 0.3 • Translate among representations

COMMON FOUNDATIONS • PAGE 4

Rules: slope = Δvertical / Δhorizontal. On a velocity-time graph, signed area represents displacement.

8. Draw v(t): 0-2 s, velocity rises linearly from 0 to 6 m/s; 2-4 s, velocity is 6 m/s.

9. From your graph, find the acceleration in 0-2 s and the displacement in 0-4 s.

Vectors vs. scalars

Scalar: magnitude only. Vector: magnitude and direction. $A = A_x i + A_y j + A^z k$.

10. Classify: displacement, speed, energy, net force. Justify one choice.

11. For A = (2, −1, 2), find |A|. Show your component calculation.

STRIDES: Draw a 3D coordinate system and a vector with nonzero i, j, k components.

Algebra II Readiness

Rearranging, quadratics, exponents & physical solutions

ALGEBRA/TRIG PATHWAY • HONORS PHYSICS + ECE1201 • PAGE 5

Rules to add: isolate the requested variable before substitution; quadratic formula; exponent rules; evaluate physical roots.

A1. Solve symbolically for a: $v^2 = v_0^2 + 2a\Delta x$.

A2. Solve for t: $0 = 20 + 15t - 5t^2$. State which solution is physical.

A3. Given $F = Gm_1m_2/r^2$, solve symbolically for r (positive separation).

A4. A model is $y(t) = 4(0.5)^t$. Determine y(0), y(1), and y(2). Describe the pattern.

> Why it matters: manipulating symbols before numbers makes unfamiliar physics questions easier to solve and makes unit-checking possible.

Trigonometry & Vector Components

Reference axes, quadrant signs, radians & graphical reasoning

ALGEBRA/TRIG PATHWAY • HONORS PHYSICS + ECE1201 • PAGE 6

Rules: $F_x = F\cos\theta$; $F_y = F\sin\theta$ when θ is measured counterclockwise from +x. radians = degrees × π/180.

A5. A 20-N force points at 150° from +x. Find F_x and F_y; explain the signs.

A6. A displacement is (−3, 4) m. Find magnitude and direction from +x.

A7. Convert 135° to radians. Convert 2π/3 radians to degrees.

A8. Draw A5 using scaled axes. Label the angle, each component, and the resultant.

Optional extension: In the 3D vector lab, try A=(3,0,0), B=(0,2,0). Use the right-hand rule to predict A×B before displaying it.

Calculus Review: Change & Accumulation

Rates, derivatives, integrals, initial conditions & units

CALCULUS PATHWAY • AP PHYSICS C/ECE1401 • PAGE 7

Rules: $v=dx/dt$, $a=dv/dt$; $\Delta x = \int v\,dt$ over the interval; include constants of integration.

C1. For $x(t)=2t^3-3t^2+5$ (SI units), derive $v(t)$ and $a(t)$.

C2. For $v(t)=3t^2$ m/s with t in seconds, find displacement from 0 to 2 s.

C3. For $F(x)=4x$ N with x in meters, calculate work from $x=0$ to $x=3$ m.

C4. If $a(t)=6t$ m/s² and $v(0)=2$ m/s, find $v(t)$.

> Concept check: A derivative describes an instantaneous rate; a definite integral represents accumulated signed change over an interval.

Separable Differential Equations

Five steps for motion with resistance

CALCULUS PATHWAY • AP PHYSICS C/ECE1401 • PAGE 8

Physical model: with horizontal velocity v and linear resistance b, m dv/dt = −bv. Assume m>0 and b>0.

Step	Action	Your work
1	Choose + direction	_____
2	Separate variables	_____
3	Integrate	_____
4	Use $v(0)=v_0$	_____
5	Interpret time scale	_____

C5. Solve with v(0)=12 m/s, m=2 kg, b=0.8 kg/s. Express v(t).

C6. Find the time when this object reaches 6 m/s. Give units.

Important: Dividing by v assumes $v \neq 0$. The equilibrium solution v=0 is also valid. Define $\tau=m/b$; its unit must be seconds.

Model, Test & Extend

Terminal velocity with linear drag • Downward chosen positive

CALCULUS PATHWAY • AP PHYSICS C/ECE1401 • PAGE 9

Model: $m\,dv/dt = mg - bv$; with $v(0)=0$ and constant $g=9.8\ m/s^2$.

C7. Show that $dv/(g - (b/m)v) = dt$, then integrate to find $v(t)$.

C8. For $m=2$ kg, $b=0.8$ kg/s, find terminal speed and time constant.

C9. Use the webpage slider: what happens when b doubles? Explain in physics and math.

C10. Sketch v versus t for a release from rest; mark v_T and τ (qualitatively).

Future connection: the differential-equation model returns in dynamics and thermal extensions. Not every second-order differential equation is separable.

Interactive 3D Vector Laboratory

Live tool at /physics/unit0/ • Predict → Observe → Explain

COMMON VECTOR LAB • ALL COURSES • PAGE 10

Tool: enter i, j, k components; drag to rotate; zoom; display dot and cross products. Turn the cross-product arrow on.

Case	A (i,j,k)	B (i,j,k)	A·B	A×B	Angle
I	(3,0,0)	(0,2,0)	_____	_____	_____
II	(0,2,0)	(3,0,0)	_____	_____	_____
III	(2,0,0)	(1,0,0)	_____	_____	_____
IV	(1,-2,3)	(4,0,-1)	_____	_____	_____

V1. What changes and what stays the same when A and B are swapped?

V2. Why does A·B=0 in Case I? Explain with geometric meaning.

V3. Why is A×B a vector? Use the right-hand rule for Case I.

> Physics links: dot product → work by a constant force; cross product → torque. No calculus required to explore the geometry.

Shared Inquiry: Two Sunsets

Use geometry, a model, and a critique - not a memorized formula

COMMON MODELING INQUIRY • ALL COURSES • PAGE 11

A person at the beach stands after the sun sets while lying down. Assume eye height h=1.70 m and elapsed time t=11.1 s. Earth rotates once in approximately 86400 s.

S1. Draw Earth, the observer, the tangent line of sight, R, h, and the horizon point.

S2. Explain why $\cos\theta = R/(R+h)$ follows from your triangle.

S3. Find $\theta = (2\pi/86400)t$ in radians, then rearrange for R.

S4. Estimate R. State the order of magnitude and two model limitations.

Model-check: an answer around millions of meters is plausible; discuss atmospheric refraction, observer reaction time, and idealized rotation/geometry.

Unit 0 Readiness Check

Show reasoning, dimensions and physical interpretations

COMMON + COURSE-SPECIFIC CHECK • PAGE 12

Common (all): Complete questions 1-3. Then complete the one question assigned to your pathway.

1. Convert 25 cm^2 to m^2 and explain the exponent used.

2. State the units of the slope of a position-time graph and what it means.

3. A=(1,-2,3) and B=(4,0,-1). Find A·B. Is the angle acute, obtuse or right?

4A - Honors Physics / ECE1201: resolve a 40-N force at 30° above +x.

4C - AP Physics C/ECE1401: solve dv/dt=-0.5v for v(0)=8 m/s; find t at v=4.

Exit reflection

Which mathematical tool or measurement habit will you revisit first, and why?

Student self-check: I can draw, calculate, verify units, interpret a graph, and explain an answer in words.

Physical Inquiry: Drop & Time

Shared hands-on investigation • Structured error analysis

PHYSICAL LAB • ALL COURSES • PAGE 13

Driving question: Can measurements of a falling object determine gravitational acceleration, and is the result consistent with a model after uncertainty is propagated?

Physical setup and safety

• Work with a partner. Drop a soft, non-breakable ball onto a clear floor from approximately shoulder height or lower. Never stand on furniture.

• Use a meterstick or tape to measure vertical distance from the object's release position to the landing surface. Record where you define the start and stop.

• Time at least 8 actual drops. Use the same release height and method; swap observer roles if your teacher directs.

• Record a stopwatch display increment, the height uncertainty, and possible bias from release and reaction timing.

Model and prediction

For a release from rest with negligible air resistance, and with downward positive:

$h = (1/2)gt^2$ and $g = 2h / t^2$.

P1. Predict the fall time for your measured height using $g = 9.81\ m/s^2$.

P2. Identify two factors that could make your measured time differ from the ideal prediction.

P3. Define the start event, stop event, and exact reference points for height.

Live web report: /physics/unit0/drop-lab/ contains the data entry, statistical uncertainty, propagation steps, interpretation prompts and print-to-PDF report. Do not substitute simulated times for physical trials.

Raw Measurements & Timing Uncertainty

Collect actual data before calculating a result

PHYSICAL LAB • ALL COURSES • PAGE 14

Measured height h: _____ m Estimated u_h: _____ m Stopwatch increment r_t: _____ s

Trial	Time t (s)	Trial	Time t (s)
1	_____	5	_____
2	_____	6	_____
3	_____	7	_____
4	_____	8	_____

Scaffolded analysis

1. Calculate mean $\bar{t} = \Sigma t / n$ and sample $s = \sqrt{[\Sigma(t - \bar{t})^2/(n-1)]}$. Show one substitution.

Your mean and sample standard deviation:

2. The uncertainty of the mean from trial-to-trial scatter is $s/\sqrt{n}$. An estimate of standard uncertainty from rounding to the stopwatch increment is $r_t/\sqrt{12}$.

Calculate $u_t = \sqrt{[(s/\sqrt{n})^2 + (r_t/\sqrt{12})^2]}$. State the assumptions.

This u_t measures trial scatter and timer quantization only. A consistent reaction-time delay or early stop can bias all trials and is not captured by the scatter. Identify that limitation in your conclusion.

Algebra & Trig: Propagate Uncertainty

Honors Physics + ECE1201 • Same physical data

PHYSICAL LAB • ALGEBRA PATHWAY • PAGE 15

Apply a provided uncertainty rule

For a calculated quantity $g = 2h/t^2$, assume independent small uncertainties in height and mean time:

$u_g / |g| = \sqrt{[(u_h/h)^2 + (2u_t/t)^2]}$

Explain the factor 2: the time is squared in the denominator, so a small relative time uncertainty affects g about twice as strongly.

A1. Compute $g = 2h/t^2$ with your actual measurements. Show units.

A2. Compute fractional height uncertainty u_h/h and timing contribution $2u_t/t$.

A3. Combine in quadrature; report $g \pm u_g$ in m/s^2, with sensible rounding.

A4. Is 9.81 m/s^2 inside your reported interval? What might explain disagreement?

> Do not report percent error as percent uncertainty: they answer different questions. The first compares to an accepted value; the second describes precision within the stated model.

Calculus: Derive the Propagation

AP Physics C/ECE1401 • Same physical data

PHYSICAL LAB • CALCULUS PATHWAY • PAGE 16

Start from the physical model

Given $g(h,t) = 2h/t^2$, small independent standard uncertainties propagate to:

$u_g^2 \approx (\partial g/\partial h \cdot u_h)^2 + (\partial g/\partial t \cdot u_t)^2$.

C1. Starting with $dv/dt = g$ and $v = dx/dt$, integrate to obtain $h = (1/2)gt^2$ for release from rest.

C2. Calculate $\partial g/\partial h$ and $\partial g/\partial t$; include dimensions.

C3. Substitute your measured h, t̄, u_h, and u_t into the derivatives formula.

C4. Factor g and show that your result agrees with the relative-uncertainty form.

Extension: estimate the first-order effect of a +0.10 s bias in reported time using $\Delta g \approx (\partial g/\partial t)\Delta t$. Why is this not the same as random uncertainty?

Lab Report & Model Evaluation

All courses • Submit through the web lab or complete in writing

PHYSICAL LAB • ALL COURSES • PAGE 17

Evaluate evidence

R1. Report g ± u_g, interval, and percentage uncertainty. Compare with 9.81 m/s².

R2. Which measurement contributes more to u_g: height or time? Use numerical evidence.

R3. Describe a systematic timing bias and predict whether it would make calculated g too large or too small.

R4. Revise one part of the procedure to reduce the dominant uncertainty. Explain how you would test the improvement.

Submission checklist

- Actual raw times (at least 8), measured height, and instrument resolution recorded.
- Mean, sample spread, uncertainty of the mean, and height uncertainty explained.
- g and propagated uncertainty shown with a valid algebraic or calculus method.
- Conclusion distinguishes repeatability, uncertainty and possible systematic bias.

> Digital option: complete the structured form at /physics/unit0/drop-lab/ and choose “Print / Save PDF report”. The printed report is the primary submission; these pages are the calculation and backup record.